Energy Loss of an Oscillating Water Column

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Abstract

We examine the effects of tube diameter on the period and decay constant of a vertically oscillating water column. Using transparent tubes with diameters ranging from 17 to 88 mm, oscillations were recorded and analyzed using video tracking. Theoretical models predict that the period is independent of both oscillation amplitude and tube diameter. The results confirm this independence, showing a constant period of approximately 1.29s. However, relative energy loss was found to increase with oscillation amplitude through a power-law relationship, with the exponent increasing with tube diameter before plateauing at a value close to 0.41; this indicates that the damping behavior becomes increasingly nonlinear as tube diameter increases, approaching a velocity dependence of drag around $v^{1.41}$ for larger tubes.

Keywords: Oscillating water column, Damping, Viscous drag, Energy loss

I. INTRODUCTION

The oscillation of fluids in cylindrical tubes is a phenomenon with applications including liquid siphons, manometers, and devices involving fluid transfer. When an open cylindrical tube is placed vertically in water, sealed at the top, then lifted and unsealed, the water column inside the tube undergoes vertical oscillatory motion about an equilibrium position.

Previous investigations have established many of the fundamental properties of oscillating water columns. Despite the presence of damping, the oscillation period remains nearly constant throughout the motion, while the upper and lower half-periods differ slightly during the early oscillations before converging as the oscillation amplitude decreases.[1]

Due to dissipative forces acting on the system, the oscillation diminishes as the oscillatory motion progresses and eventually ceases. Understanding the mechanisms responsible for this damping is important not only for modeling real fluid systems but also for extending the framework of simple harmonic motion (SHM) to non-ideal, viscous environments. In an ideal SHM system, a restoring force proportional to displacement acts on the oscillating body, producing a constant period independent of amplitude. However, real systems experience energy loss due to resistive forces, resulting in damped oscillations characterized by a decay constant.

In the case of a water column oscillating within a tube, damping arises in part by viscous drag between the water and the inner walls of the tube.[2] Viscosity describes a fluid's resistance to deformation and flow, while drag forces oppose motion through a medium and depend on the area of contact between the moving fluid and the solid boundary.[3] Since this contact area depends directly on the radius of the tube, tube radius is expected to influence the magnitude of damping experienced by the oscillating water column. The surface area, A, of water in contact with the inner wall of a cylindrical tube is given by:

$$A = 2\pi rh \tag{1}$$

where r is the inner radius of the tube, and h is the height of the water column. As the radius increases, the contact area, and therefore the viscous drag, also increases. This suggests that tube diameter will influence the rate at which oscillations decay, quantified by the decay constant

As wall area decreases, wall-bound viscous shear becomes larger relative to the inertial mass. Given that fluid velocity at the tube wall is zero due to the "no-slip" condition, in narrower tubes, the viscous boundary layer occupies a larger percentage of the total cross-sectional area.[4] This creates a greater velocity gradient and higher shear stress, leading to greater energy dissipation through viscous resistance.

For viscous damping, viscous drag force is expected to be proportional to fluid velocity, thus simple harmonic oscillations with viscous drag experience exponential decay for both oscillation amplitude and system energy. The relative energy loss per period, ΔE_R, would therefore be constant and independent of oscillation amplitude:

$$\Delta E_R = kA^0 \tag{2}$$

where A is the oscillation amplitude and k is a constant, thus for systems where viscous drag is the dominant factor, ΔE_R is constant for all amplitudes.

Although wall-induced viscous drag contributes to energy dissipation in oscillating water columns, it is not the dominant mechanism for large oscillations of large-diameter tubes containing low-viscosity fluids. As water enters the tube during the upward motion, the abrupt constriction from the large reservoir into the much narrower tube produces turbulent vortices near the tube entrance. The formation of these vortices converts part of the fluid's kinetic energy into turbulent motion, reducing the mechanical energy of the oscillating water column.[5]

During the downward motion, water exits the tube as a jet into the surrounding reservoir. The kinetic energy carried away by this jet is effectively lost from the oscillating column because it is dissipated within the much larger body of water outside the tube. These entrance and exit losses occur during every oscillation and become particularly significant when fluid velocities are high. Viscous drag along the tube walls continues to dissipate energy throughout the motion, but for the relatively large tube diameters used in this investigation, previous experiments demonstrated that these viscous effects become dominant only after many oscillations, once the viscous boundary layer has fully developed.

A theoretical model was developed by Lorenceau et al describing oscillating liquid columns in which the dominant source of damping during the early stages of motion is not ordinary viscous friction, but singular pressure losses associated with the entrance and exit of the fluid.[5] Unlike the exponential damping observed in many simple harmonic oscillators, the model predicts that the oscillation amplitude decreases approximately hyperbolically with time because the energy loss depends on the instantaneous velocity of the fluid rather than remaining a constant fraction of the system's energy. Consequently, the relative energy loss is expected to increase proportionally with oscillation amplitude, indicating that the dominant resistive forces are nonlinear.

This nonlinear behavior contrasts strongly with the damping observed in systems such as oscillating U-tubes, where the flow remains predominantly laminar, and the drag force is approximately proportional to velocity. In such systems, the oscillation amplitude decreases exponentially, producing an approximately constant relative energy loss during successive oscillations.[6] In the open vertical tube, however, the repeated formation of entrance vortices and the expulsion of water into the surrounding reservoir produce turbulent losses lead to effective damping forces that are expected to scale approximately with the square of the fluid velocity. Since the maximum velocity of the oscillating column is proportional to its oscillation amplitude, Lorenceau et al predict that the relative energy loss will increase approximately linearly with oscillation amplitude.[5]

This investigation examines these predictions experimentally using five transparent tubes of different inner diameters. First, the experiment confirms the previously established observations that the oscillation period remains essentially independent of amplitude and tube diameter and that the half-period asymmetry is also observed. The primary objective of this study, however, is to investigate the nature of the energy loss responsible for damping. Specifically, the relationship between relative energy loss and oscillation amplitude is examined to determine whether the experimental results support the prediction of nonlinear damping. In addition, the dependence of the proportionality constant relating relative energy loss to oscillation amplitude on tube diameter is investigated to determine how tube diameter influences the dominant energy-loss mechanisms in oscillating water columns.

II. METHODS

Five 80 cm long cylindrical plastic tubes with inner diameters ranging from 17 to 84 mm were used. Each tube was submerged vertically in a transparent 35 cm diameter water tank filled with tap water. A lid with a silicone seal was placed over the top of the tube while it was pushed downward 40 cm below its equilibrium position, as shown in figure 1. The lid was then released, allowing water to rise into the tube and undergo damped vertical oscillations about its equilibrium height. Five trials were performed for each tube diameter.

A meter stick was mounted immediately adjacent to the tube to provide a length scale for position

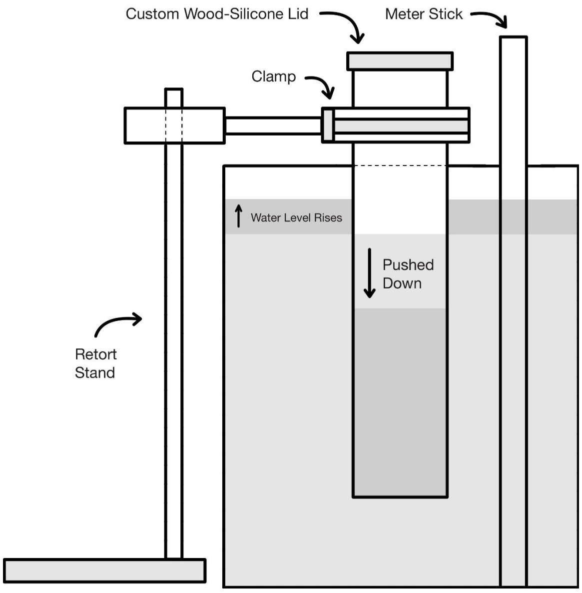

Figure 1. Diagram of the lab set-up

measurements. The inner diameters of the tubes were measured using digital Vernier calipers with a resolution of ±0.01 mm. The oscillatory motion of the water column was recorded using a smartphone mounted on a tripod to ensure that the camera remained stationary throughout each trial.

The video recordings were analyzed using Logger Pro, which allowed the vertical position of the water surface to be tracked frame-by-frame as a function of time. A selection of representative frames from the video is shown in Figure 2.

From the displacement versus time graphs produced by Logger Pro, the times and amplitudes of successive oscillation peaks were extracted. These data were used to calculate the oscillation period, half-periods, and the ΔER between successive oscillations.

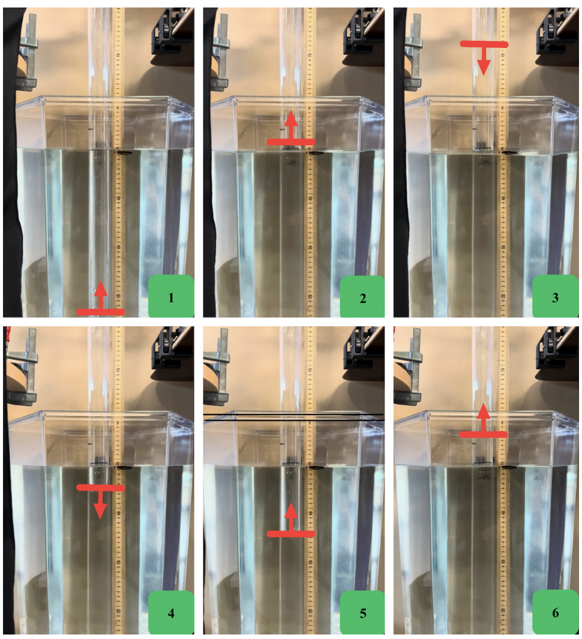

Figure 2. Photos of water column oscillation

The ΔE_R was determined from the amplitudes of two consecutive peaks:

$$\Delta E_R \propto \frac{(A_n)^2 - (A_{n+1})^2}{(A_{n+1})^2} \qquad (3)$$

where A_n is the amplitude of the n^{th} oscillation peak and A_n is the amplitude of the subsequent peak. The above relationship represents the fraction of oscillation energy lost during one cycle relative to the previous amplitude.

Although the primary objective of this investigation was to examine the nature of the energy loss, the oscillation period was also measured for each trial to verify the previously established result that the period is independent of oscillation amplitude and tube diameter. The average oscillation period for each tube diameter was determined by calculating the mean period for six consecutive oscillations.

III. RESULTS & DISCUSSION

The oscillation period remained approximately constant throughout each trial despite the gradual decrease in oscillation amplitude, and no significant dependence of the period on tube diameter was observed. The mean measured period was 1.290 ± 0.006 seconds. Small differences between the top and bottom half-periods were present during the initial oscillations but became progressively smaller as the amplitude decreased, as shown by Chung.[3] Since these results confirm previously established observations, they are not discussed further and instead (4)serve as validation of the experimental procedure.

The principal objective of this investigation was to examine the mechanism responsible for the loss of energy during each oscillation. To investigate this, the ΔE_R was plotted against oscillation amplitude. Figure 3 shows the variation of ΔE_R with amplitude for the 17.22 mm diameter tube. The relationship is nonlinear, with ΔE_R increasing with amplitude at a progressively decreasing rate. The fit for the 17.22mm tube is:

$$\Delta E_R = 1.18(Amplitude)^{0.32} \qquad (4)$$

The exponent B = 0.321 indicates that ΔE_R increased with amplitude, but at a lower than proportional rate. If the model of damping dominated by singular pressure losses proportional to v^2 were applicable,

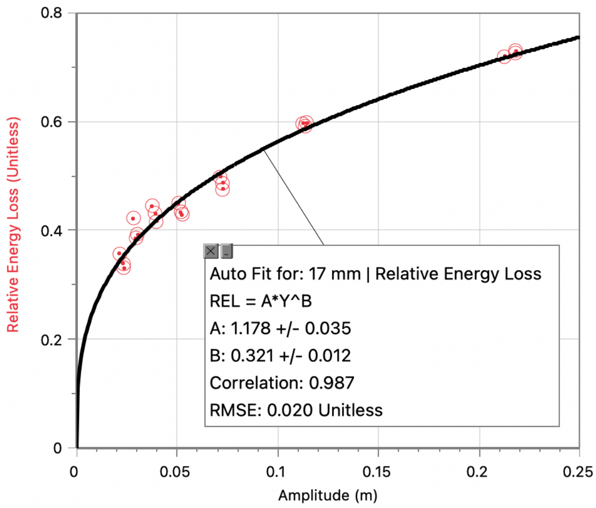

Figure 3. The variation of ΔE_R with amplitude for the 17.22mm diameter tube

ΔE_R would be expected to be proportional to amplitude, and exponent B would be close to 1.[5]

The B value of 0.321 therefore indicates that Lorenceau's model does not describe the behavior of the 17.22 mm tube. However, B is also not equal to zero, which excludes purely viscous damping. The coefficient A = 1.178 is related to the magnitude of ΔE_R as a function of amplitude. Similar ΔE_R versus amplitude power fits were obtained for all five tube diameters. The values of the coefficient A, and exponent B are shown in Table 1.

The values in Table 1 were obtained from power fits of ΔE_R against amplitude for each tube, similar to that shown in Figure 3. The coefficient A increased from approximately 1.18 to 1.28 as tube diameter increased, indicating that for a given amplitude, larger diameter columns experience a greater magnitude of ΔER. This likely means the larger water columns produced greater displacement of surrounding water at the open end of the tube, resulting in greater energy dissipation due to sloshing motion.

Diameter (±0.01 mm)	**Coefficient A (mm^{-1})**	**Exponent B (unitless)**
17.22	1.18 ± 0.04	0.32 ± 0.01
32.99	1.21 ± 0.04	0.37 ± 0.01
43.62	1.24 ± 0.04	0.39 ± 0.01
53.5	1.26 ± 0.08	0.41 ± 0.03
84.4	1.32 ± 0.07	0.42 ± 0.02

Table 1. Power-fit data for the relationship between ΔE_R and oscillation amplitude for each tube diameter

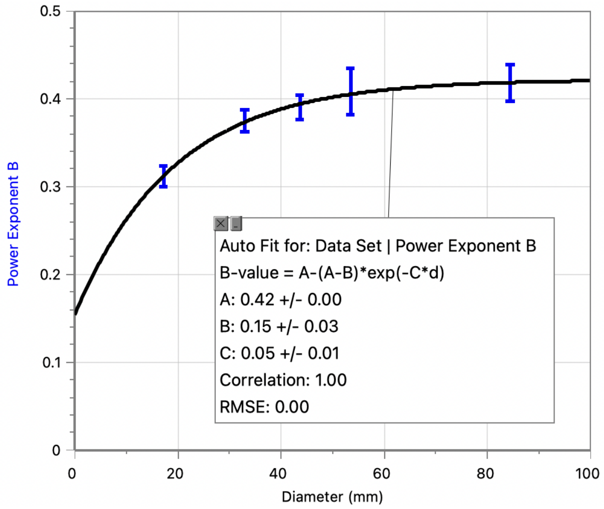

Figure 4. The variation of the power exponent B with tube diameter.

The exponent B increased from 0.32 for the 17.22 mm tube to around 0.41 for the two largest tubes. This indicates that the effective damping force increases from $F_d \propto v^{1.32}$ for the 17.22 mm tube to around $F_d \propto v^{1.4}$ for tubes greater than 50 cm diameter, again showing that the effective damping force has a velocity dependence that is greater than what would be expected for viscous drag, but not close to the dependence predicted by Lorenceau et al.

The overall shape of the exponent value trend (Figure 4) shows an initial increase with tube diameter followed by a plateau near B = 0.41. One possible explanation is that Lorenceau's model is overly simplified, and thus is not applicable to this situation. His model assumed an infinite reservoir where the water level outside the tube remains unchanged. The tank for this experiment was approximately 35 cm wide, and the water level in the tank rose and fell noticeably during the oscillations in the larger tubes. These movements may reduce the relative velocity of water in the oscillating water column and the surrounding reservoir, potentially reducing the energy loss compared to an infinite reservoir.

Models of unsteady oscillatory flow indicate that the effective energy loss coefficient and drag behavior can depend on the velocity regime and may therefore produce a velocity dependence around $v^{1.4}$ depending on the aspects of the behavior considered.[7] However, these interpretations remain speculative. Further research involving larger tanks or larger tube diameters is needed to determine whether the asymptote near B = 0.41 is caused by unsteady vortex dynamics, finite-reservoir effects, or other damping mechanisms.

IV. CONCLUSION

It was shown that tube diameter has no significant effect on the oscillation period or half-period difference, consistent with predictions. Observing the relationship between ΔE_R and oscillation amplitude, however, showed that the damping becomes increasingly nonlinear as tube diameter increases, reaching a maximum damping force proportional $v^{1.4}$ for larger diameter tubes. The results support neither the model proposed by Lorenceau et al nor models that propose purely viscous damping, suggesting that complex behaviors determine the energy loss.

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