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Temperature Dependence of the Coefficient of Restitution for a Rubber Ball

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Abstract

The relationship between the temperature and the coefficient of restitution of a polybutadiene superball was investigated. The ball was released from a fixed height and tracked with a motion detector as it bounced. The coefficient of restitution at temperatures ranging from 223K to 341K was determined. It was found that the relationship was inversely exponential, approaching a maximum above 300K.

Introduction

The speed with which a ball leaves the floor is always less than the impact speed due to energy losses occurring during the deformation of the ball while it is contact with the floor. It is known that the temperature of rubber affects its stiffness [1], so it is expected that temperature will have an effect on the 'bounciness' of a superball.

Coefficient of restitution for a ball bouncing on a massive floor is the ratio of the relative velocities before and after impact.

$$C_R = \sqrt{\frac{h_b}{h_i}} \qquad \text{(Equation 1)}$$

where C_R is the coefficient of restitution, h_b is the maximum height after the bounce, and h_i is the initial height [2]

Hysteresis is related to the energy dissipated as heat in a material in one cycle of contraction and extension. Hysteresis occurs in rubber balls when the ball is dropped onto the floor because rubber does not obey Hooke's Law perfectly. This can be seen by a hysteresis loop as shown in Figure 1. The area under the blue line represents the energy used to compress the ball while the area under the red line represents expansion after contact with the floor. The energy needed to compress the ball is greater than energy released during expansion. The energy lost is dissipated as heat, represented by the area of the hysteresis loop.[3]

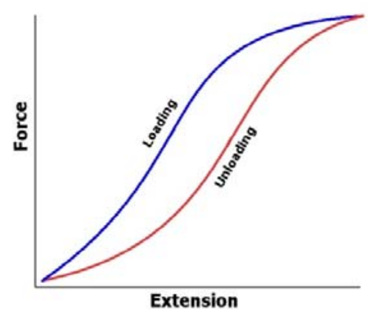

Figure 1 Idealized graph of elastic hysteresis for rubber. [3]

It is known that at high temperatures, heat production due to hysteresis of rubber for a given deformation is reduced [4]. Reduced heat production results in an increase in the coefficient of restitution of the rubber ball.

Young's modulus is the measure of the stiffness of an isotropic elastic material. Since Young's modulus for rubber is roughly proportional to temperature (T) [1], an increase in temperature of the rubber will result in an increase in Young's modulus, or in other words, a decrease in the compression distance of the ball. Less energy will be dissipated in each bounce, leading to an increase in the coefficient of restitution.

Given these two known relationships, the coefficient of restitution is expected to increase as the temperature increases.

Methods

The polybutadiene superball used, Figure 2, had a diameter of 4.8 ± 0.1cm, and a mass of 52.51 ± 0.01 g. The ball was released from a height of 0.860 ± 0.003 m beneath a fixed motion detector, as shown in Figure 3. A thread was attached to the ball, and the ball held by this to reduce any spin upon release.

Figure 2 The rubber ball used.

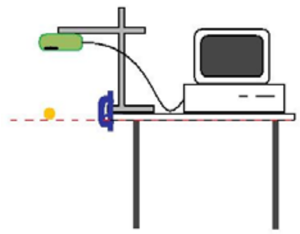

Figure 3 Illustration of the set-up.

The position of the ball was tracked as it fell and bounced on a hard, linoleum-covered concrete floor, as shown in figure 4. The coefficient of restitution was determined from this data.

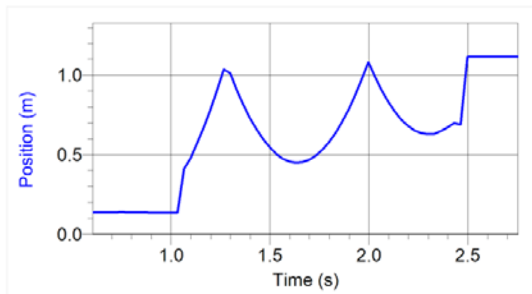

Figure 4 The height of the ball after one bounce was determined. The position represents the distance from the motion detector to the top of the ball.

The temperature of the ball was changed by submerging it in a liquid bath for 10 minutes before each set of trials. A dry ice and ethanol bath was used for the low temperatures and a water bath was used for high temperatures. Temperatures tested ranged from 223K to 341K, with three trials taken at each temperature.

Results and Discussion

From figure 5, the relationship between temperature of the ball and the coefficient of restitution is given by the equation

$$C_R = -6000e^{-0.044} + 0.90 \qquad \text{(Equation 2)}$$

There is an inverse exponential relationship between the temperature and the coefficient of restitution.

The coefficient approaches a maximum value of 0.90. Even when the ball is at high temperatures, some of the mechanical energy of the ball will be lost during each bounce. Hysteresis is less significant at higher temperatures but does not fully disappear.

The minimum coefficient of restitution found during this experiment was around 0.56, at 223K, as this was the coldest our ethanol bath could reach. Given that only one temperature between 200 K and 250 K was tested, there is little confidence in equation 2 at temperatures below 250 K. Further tests could be conducted at temperatures between 200K and 250 K.

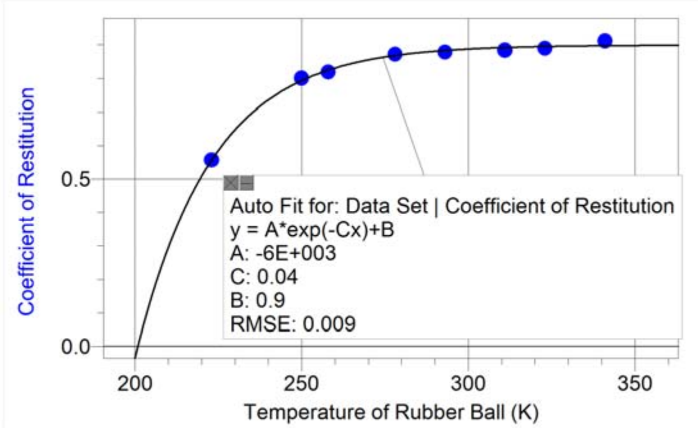

Figure 5 The temperature of the rubber ball is plotted against the coefficient of restitution. An inverse exponential relationship is shown.

The relationship between the coefficient of restitution and temperature for situations using a different ball or surface is expected to result in a similar inverse exponential relationship, with different maximum coefficients at high temperatures. This is expected since all rubber materials exhibit hysteresis.

Conclusion

There is an inverse exponential relationship between the temperature of the polybutadiene superball bouncing on a linoleum covered concrete floor and the coefficient of restitution. The coefficient approaches a maximum of 0.90, at temperatures above 300 K.

References

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