ISB Journal of Physics, Vol. 1, Iss. 1 June 2007
http://www.isjos.org

Coefficient of Restitution of a Tennis Ball

Andre Roux, Jennifer Dickerson
IB Physics, International School Bangkok

Abstract: The coefficient of restitution (COR) of a tennis ball was investigated over a range of impact velocities. It was found that the COR of the ball was lower than ATP regulations specify, and that the COR decreased with increasing impact velocity.

Introduction

The governing bodies of all modern sports carefully define the specifications of the equipment which may be used in the sport. This is to ensure that all tournaments are played under standard conditions and that all players compete with standard equipment. The ATP has defined the specifications for tennis balls which may be used in official tournament play. One important characteristic of a tennis ball is its "bounciness". According to ATP regulations, a tennis ball must bounce to a height of between 135 cm and 147 cm when dropped from 254 cm on to a hard surface.[1]

In physics, the "bounciness" of a ball is defined as the coefficient of restitution. The coefficient of restitution (COR) for a ball bouncing off a fixed surface is defined as the ratio of the velocity of a ball after it bounces to the velocity of the ball before it bounces.

$$COR \equiv \frac{v'}{v_0}, \qquad (1)$$

where v_0 is the initial velocity and v' is the velocity after the bounce.[2]

An object with a coefficient of restitution of 1.0 will have no loss of speed after it bounces. If it is released from a certain height, that object will fall and bounce back up to the exact same height. An object with a COR of 0.0 will hit the ground and not bounce at all.

The COR for regulation tennis balls can be calculated from the ATP regulations. Ignoring the effects of air resistance, a ball dropped from a height of 254 cm will have a velocity of 7.06 m/s just before it hits the ground. According to the regulations, the tennis ball must then bounce to a height of between 135 cm and 147 cm, meaning the ball must have a velocity of between 5.14 m/s and 5.36 m/s as it leaves the ground. This means that, ignoring the effects of air resistance, a regulation tennis ball would have a COR of between 0.728 and 0.759. Taking into account the effects of air resistance, the actual COR of a tennis ball would be larger than this.

The ATP has defined the requirements for a ball dropped from a height of 254 cm, hitting the ground at a speed of 7.06 m/s. But during a game of tennis, the ball bounces off the ground or the racquet at a large range of velocities. One wonders if the COR is constant at all velocities. This research will therefore attempt to determine how the COR is related to the initial velocity of a tennis ball for a range of velocities.

When a tennis ball strikes a surface, the rubber and fabric shell of the ball is deformed. Some of the kinetic energy of the ball is converted to thermal energy during this process. Since a higher initial velocity will cause a greater ball deformation, it can be predicted that higher initial velocity will lead to a higher proportion of energy being lost, and thus a lower COR. Since a lower initial velocity will lead to less ball deformation, it is predicted that the COR will approach one as the initial velocity approaches zero. It is predicted that the relationship between the COR and the initial velocity (v_i) is:

$$\text{COR} = 1 - Av_i^B \qquad (2)$$

where A and B are positive constants. Not enough is known about the characteristics of tennis ball deformation during a bounce to predict the values of A and B with any confidence.

Methods

A small protected area was set up in which the tennis ball could be hit without fear of damage to property or personnel.

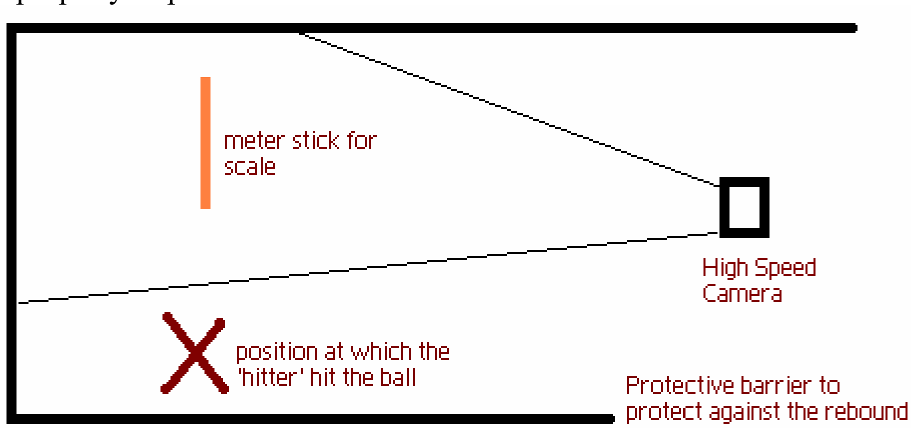

Figure 1: The figure above is an illustration of the setup used.

It was important to make sure that the tennis ball was clear on the video, so that its position can be accurately determined. A trial was conducted to make sure that the ball could be seen clearly on the digital video. When checking the field of vision of the camera it was ensured that the area used as the 'rebound area', as well as the meter stick that was placed on the ground, was completely visible in the camera. This ensured that the ball and the reference meter could be seen before, during and after the rebound.

The data recording process required at least two people, one to hit the ball against the wall with the racquet and one to stop and start the video recording. The 'hitter' stood with the racquet and the ball between 3 and 4 meters away from the 'hit-zone', ready to hit the ball against the wall when the 'recorder' gave the word. Once the 'recorder' gave the word, the video capture was started and the 'hitter' hit the ball at the wall. The 'hitter' made sure to hit the ball directly above the meter stick to ensure that the scale would be as accurate as possible. The 'recorder' then stopped the video capture after the ball hit the wall. After each trial the video was saved and checked to see if the ball was visible throughout the whole trail. This process was then repeated 48 times for initial velocities ranging from 2.4 m/s up to 11.6 m/s.

Results

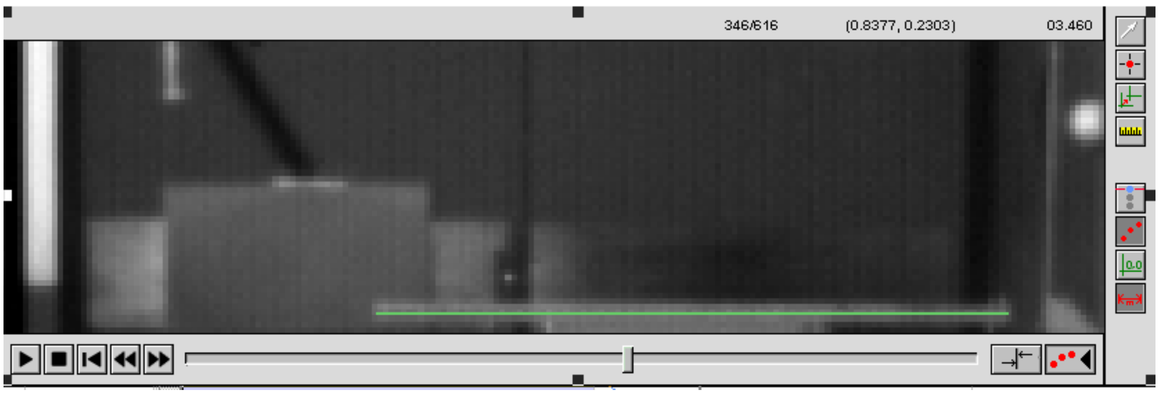

Figure 2: The image above shows the tennis ball hitting the wall on the right with the meter stick used as a scale at the bottom of the image (the green line).

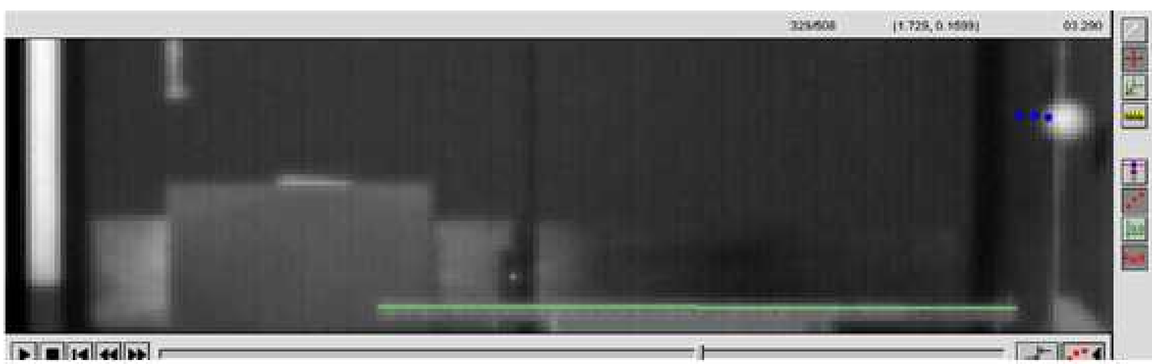

Figure 3: The image above shows the velocity vs. time graph, as well as a frame from the video showing the trail of the position of the center of the ball.

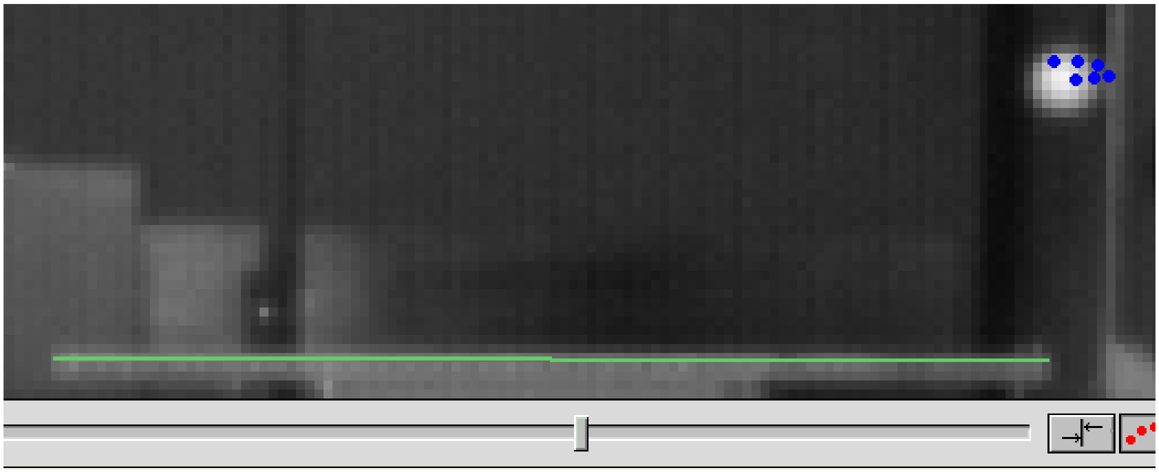

Figure 4: A frame from the video after the ball hit the wall. Again 3 data points were taken of the position of the center of the ball from the frames after the bounce.

Calculation of Velocity

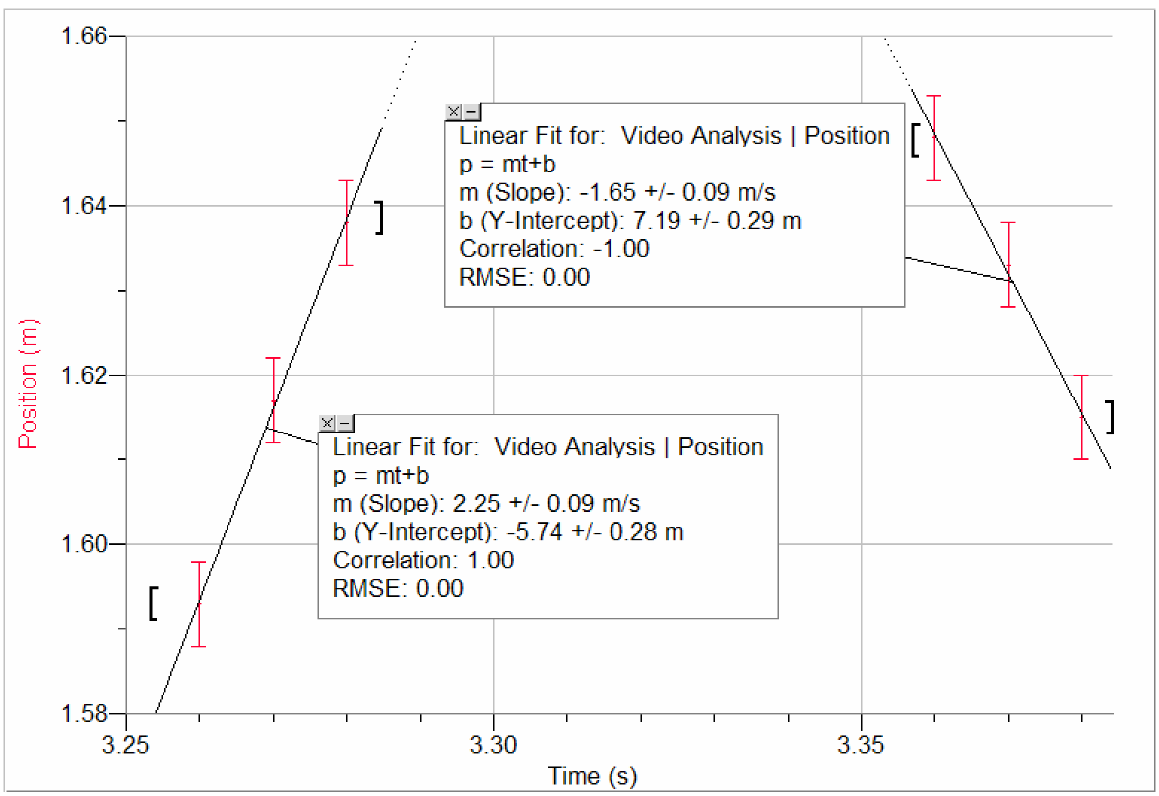

Figure 5: The graph above shows the data from Trial 1 for the velocity of the ball before and after it hit the wall. The initial and final velocities with uncertainties are shown in the analysis boxes. The uncertainty in position is estimated from the video.

Calculation of Coefficient of Restitution:

Equation: $COR \equiv \frac{v'}{v_0}$

Example from Figure 5:
1.65 *m/s* / 2.25 *m/s* = 0.73 +/- 0.06

Table 1: Velocities and Coefficient of Restitution

Trial	Initial Velocity (m/s) (+/- 0.09	Final Velocity (m/s) (+/- 0.09)	COR (+/- 0.06)
1	2.25	1.65	0.73
2	2.40	1.74	0.73
3	2.40	1.53	0.64
4	2.40	1.75	0.73
5	2.41	1.75	0.73
6	2.72	2.07	0.76
7	3.16	2.18	0.69
8	3.28	2.19	0.67
9	3.60	2.18	0.61
10	3.95	2.63	0.67
11	4.05	2.52	0.62
12	4.14	2.51	0.61
13	4.21	2.52	0.60
14	4.37	2.95	0.68
15	4.58	2.94	0.64
16	4.70	2.52	0.53
17	4.72	2.74	0.58
18	4.90	2.94	0.60
19	5.01	2.94	0.59
20	5.45	3.16	0.58
21	5.52	3.14	0.57
22	6.32	3.38	0.53
23	6.59	3.56	0.54
24	6.63	3.26	0.49
25	6.66	3.38	0.51

Trial	Initial Velocity (m/s) (+/- 0.09	Final Velocity (m/s) (+/- 0.09)	COR (+/- 0.06)
26	6.94	3.58	0.52
27	7.13	3.62	0.51
28	7.19	3.60	0.50
29	7.44	3.83	0.51
30	7.55	3.28	0.43
31	7.57	3.73	0.49
32	7.64	3.60	0.47
33	7.64	3.71	0.49
34	7.84	4.25	0.54
35	7.86	3.60	0.46
36	7.88	3.83	0.49
37	7.89	3.62	0.46
38	8.50	4.19	0.49
39	8.66	4.50	0.52
40	8.70	3.80	0.44
41	9.29	4.31	0.46
42	9.34	4.07	0.44
43	9.72	4.37	0.45
44	9.89	4.29	0.43
45	10.05	4.67	0.47
46	10.28	4.22	0.41
47	10.94	4.60	0.42
48	11.58	4.89	0.42

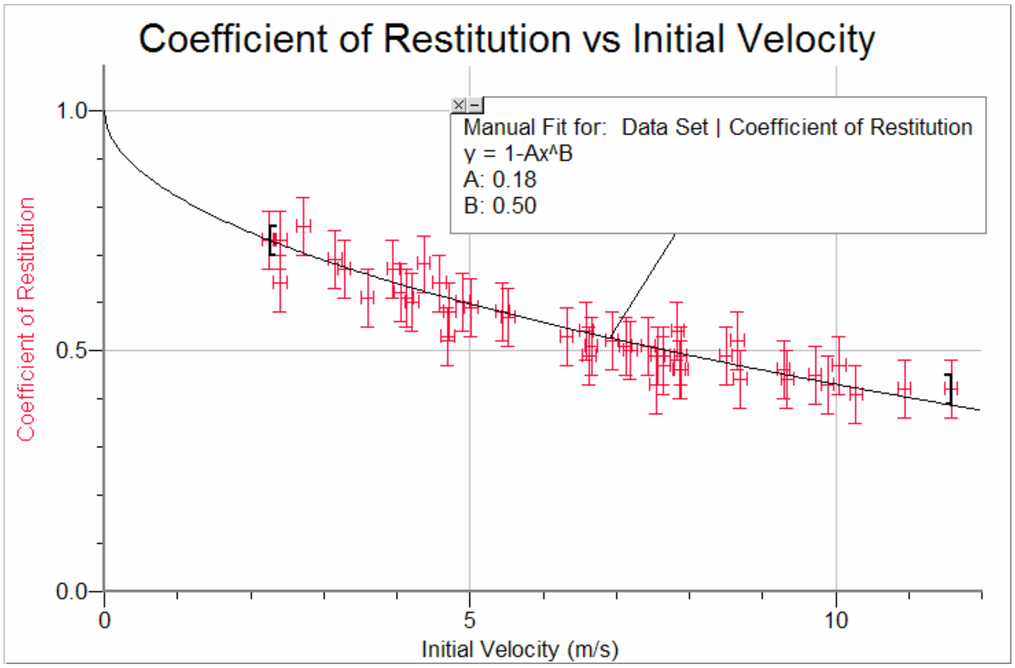

Figure 6: The graph above shows the relationship between the initial velocity and COR. Notice the clear decrease in COR over the velocity range between 2 and 12 m/s. A curve of the form predicted fits the data within uncertainties.

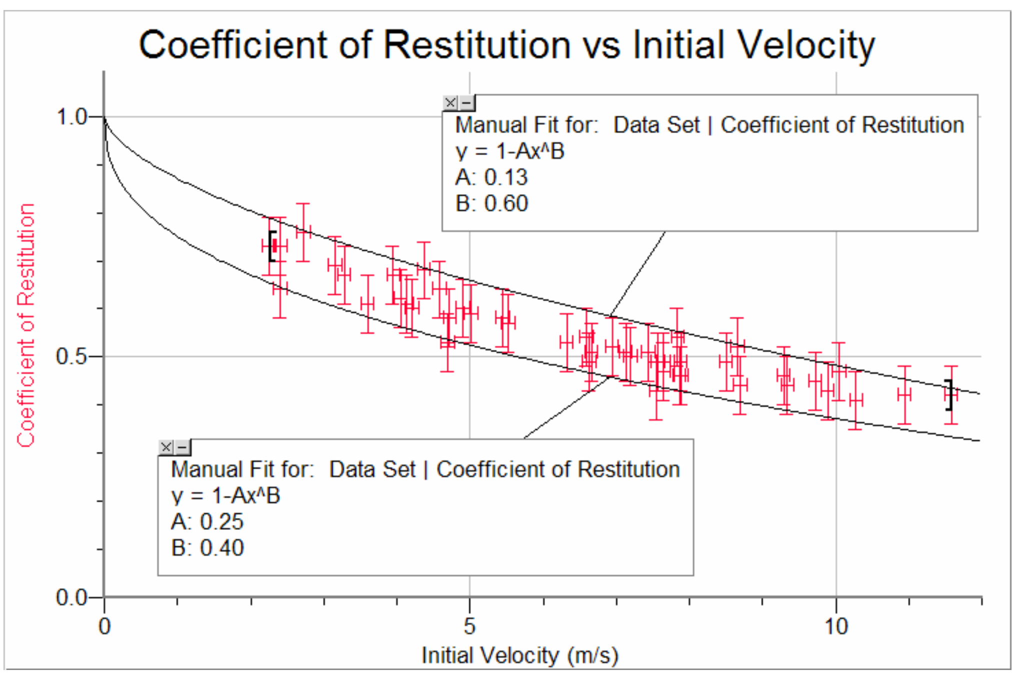

Figure 7: The graph above is the same as the one in figure 6 except with curves fitted to the extremes of the data to estimate the uncertainties in the relationship.

Conclusion and Evaluation

The relationship between the Coefficient of Restitution and the initial velocity of a tennis ball has been shown to be

$$COR = 1 - (0.18 \pm 0.07)\,(v_i)^{(0.5 \pm 0.1)}\ . \qquad (3)$$

The relationship predicted in hypothesis has been supported. While the prediction was supported, and an equation was derived, the conclusions were not strongly supported, as the uncertainty in the data and the curve fits are very large. The uncertainty in the constants of the derived equation is as much as 40% for the coefficient, and 20% for the exponent, leaving very little confidence in our conclusion.

One point to note is that the COR was significantly less than regulations require. At 7 m/s the COR was only 0.5. It was only when the ball's initial velocity was as low as 2.5 m/s that the COR increased to the regulation 0.75. This could have been due to the fact that a "fresh out of the can" ball was not used, and tennis balls are known to lose "bounciness" after they have been taken out of the can.

The one major problem with the experiment was the quality of the data obtained. The high-speed camera available had limited resolution, and the field of vision was too large, reducing the precision of the position data in each frame. In future, a camera with greater resolution should be used, and the field of vision should focus as narrowly as possible on the point of impact.

Another weakness is the nature of the tennis ball itself. A tennis ball does not have a perfectly smooth surface. The seams of the ball cause the nature of contact with the wall to be different every time, possibly contributing to variation in the results. It was one of the random errors of the experiment, one that would be difficult to control. Another random error was the method with which the tennis ball was hit against the wall. Although the racquet can provide a fair range of speeds, the location at which the tennis ball hits the wall is different every time. This is important since the reference scale needed to be at the same distance from the camera as the ball to ensure accurate data. One suggestion for improved results would be to have a tennis ball machine to shoot the ball at the exact same spot on the wall at different speeds. This would reduce uncertainties in measurements.

Further research should be conducted investigating the COR of a tennis ball over a wider range of impact velocities. Investigations into the effect of ball age/use on the COR for all major manufacturers is also recommended.

Sources:
1. http://en.wikipedia.org/wiki/Tennis_ball
2. http://scienceworld.wolfram.com/physics/CoefficientofRestitution.html