Air Pressure and Coefficient of Restitution of a Ball

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Abstract

The relationship between the pressure of a ball and its coefficient of restitution was investigated, a rubber handball was inflated to eight varying air pressures and then dropped from a constant height. With measurements of its mass, circumference, dropped height and rebound height in conjunction to the equation of coefficient of restitution, the relationship between the pressure of the ball and its coefficient of restitution was determined to be inversely exponential, with the coefficient of restitution approaching a limit as pressure increased.

Introduction

In major sports league such as NBA and NFL, there are strict rules regarding the air pressure of the ball, for the rebound of the ball varies depending on how inflated the ball is – but what is the correlation between the air pressure in a ball and how well it bounces? In this research, the relationship between the air pressure of an inflated rubber ball and the coefficient of restitution was investigated. Coefficient of Restitution (C_R) is the ratio of relative velocity after an object-surface collision. For an object bouncing off a stationary object, which is the floor in this case, the equation of coefficient of restitution is[1]:

$$C_R = \sqrt{\frac{H_2}{H_1}} \tag{1}$$

Where C_R is the coefficient of restitution, H_1 is the initial height of the object, and H_2 is the height of the object after the bounce under the assumption that the potential energy and the kinetic energy of the object are equal and there is negligible air resistance.

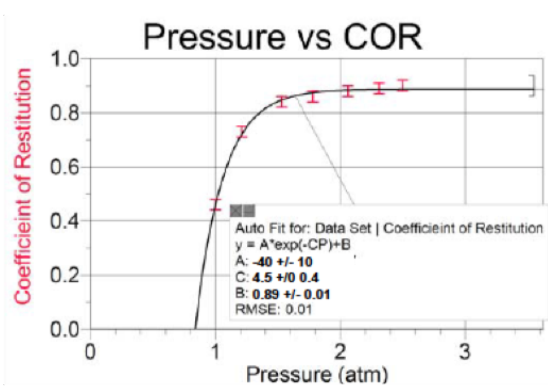

Figure 1 The graph of pressure against the coefficient of restitution by Osman and Kim.

There is currently no widely accepted equation that describes the relationship between the air pressure of an inflated ball and its coefficient of restitution. However, Osman and Kim investigated the same relationship with a different ball and proposed the equation[2]:

$$C_R = (-40 \pm 10) * e^{(-4.5 \pm 0.4)P} + (0.89 \pm 0.01) \tag{2}$$

Where C_R is the coefficient of restitution and P is the air pressure of the ball in atmospheres.

As shown in Figure 1, an inverse exponential relationship was proposed. However, as there are not many data at low pressure, the level of confidence of this model is low at lower pressures. Therefore, to increase the level of confidence in the model at low pressures, the investigation focused on the coefficient of restitution of the ball at lower pressures.

Methods

A Vernier Software Motion Detector (40 samples per second) was placed 1.20 ±0.02 m above the hard floor. A string was attached to a ball so the ball could be dropped without any spin. A bicycle pump was used to pump up the ball until it massed 369.94 ±0.01g. The ball's circumference was measured twice by wrapping a piece of white yarn along a straight crevice of the ball's surface and measuring the length with a meter stick.

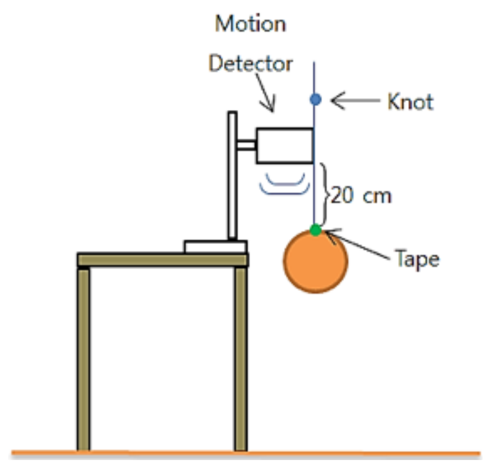

Figure 2 Apparatus Setup.

The ball was then held between the detector and the ground at a distance of 0.200 ±0.005m as shown in Figure 2 and dropped. Data was collected from before the ball was dropped to after the second bounce. After four trials, air was let out of the ball for seven other masses, for which the procedure was repeated. Finally, the ball was cut open and the thickness of the skin of the ball was measured three times in different locations using a Vernier Caliper. Temperature was kept constant at 25 ±1 degrees Celsius.

Air approximates an ideal gas when the temperature is around 300 K and the pressure is between one to three atmospheres. Thus, the ideal gas law was applied to determine the pressure of the ball. The equation of state for an ideal gas is[3]:

$$PV = nRT \tag{3}$$

Where P is the pressure of the gas, V is the volume of the gas, n is the number of moles in the gas, R is the universal gas constant (8.314 $JK^{-1}kg^{-1}$), and T is the temperature of the gas in Kelvin. Using the measured mass, circumference and the mass of the skin of the ball, the air pressure of the ball at each mass was calculated. The thickness of the skin of the ball was taken into account when determining the volume. The tested pressures in the ball ranged between 1 and 2.5 atmosphere.

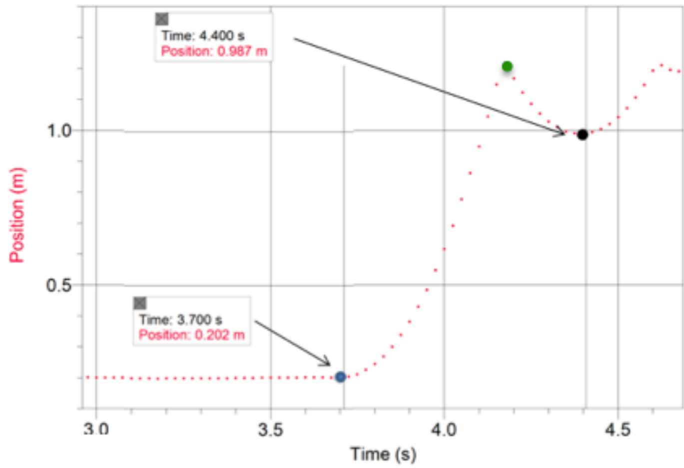

Figure 3 The position-time graph of the bouncing ball.

In Figure 3, showing the position of the bouncing ball over time, the green data point is the distance from the motion detector when the ball just hit the ground. The difference of green and blue data points is the initial height, and the difference of green and black data points is the rebound height of the ball. These data were used to calculate the coefficient of restitution.

Results and Discussion

According to Figure 4, the relationship between the air pressure of a ball and its coefficient of restitution is:

$$C_R = (-1000 \pm 900) * e^{(-8.0 \pm 0.7)P} + (0.86 \pm 0.01) \quad (4)$$

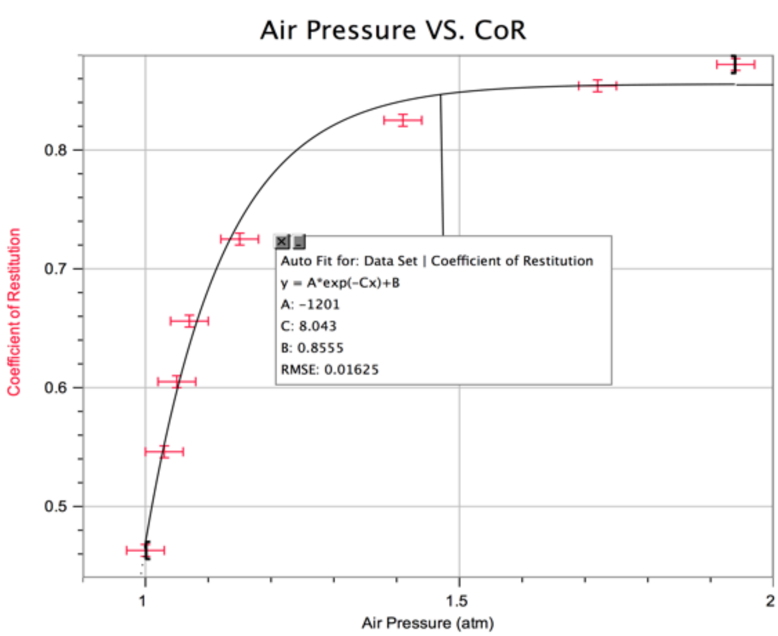

Figure 4 The graph of the air pressure against the coefficient of restitution of the rubber ball with the limits of uncertainty.

Where C_R is the coefficient of restitution and P is the air pressure of the ball in atmospheres.

This equation supports the model of the inverse exponential relationship proposed by Osman and Kim – as the air pressure increases, the coefficient of restitution also increases with an inverse exponential relation until it reaches a limit. In Equation 4, while the A-constant on the graph and the C-constant determine the curvature of the graph, the B-constant, which is 0.86±0.01, represents the horizontal asymptote of the graph. The coefficient of restitution of the tested ball would not increase more than 0.86±0.01 regardless of the pressure in the ball. This constant, of course, would vary depending on type of ball, surface it is bounced off, temperature, gas inside the ball and many other factors.

The range of data is limited to some extent because this investigation did not deal with the air pressure that is higher than 2.0 atm. However, based on the previous research of Osman and Kim, it can be predicted with confidence that even with data at higher pressure, there is no change in general trend. Considering the similar horizontal asymptote (0.89±0.01) found and similar type of ball used in the Osman and Kim paper, the theory that the coefficient of a ball reaches maximum of less than 1 and levels off is considered reliable. However, all balls are expected to have a different set of constants, since they are made out of different materials and therefore have varying bounce characteristics.

It must be recognized that the method of using a string to measure the circumference of the ball may have caused a systematic error. Since the string used in this experiment was elastic, when measuring the length of the string with a ruler, the measured length could have been greater than the actual length if the string was stretched by the force of hands holding it. To increase the level of confidence in this model, it is suggested to use an instrument that can measure the pressure in the ball directly instead of using measurements to calculate the air pressure of the ball.

Further research could investigate how a variety of common sports balls behave, or how different types of gas inside the ball would affect the coefficient of restitution.

Conclusion

The relationship between pressure of a ball and its coefficient of restitution stands to be inversely exponential, modeled by Equation 4 for pressures ranging from 1 to 2.5 atm. It is expected that the model proposed in this paper is applicable to any type of a hollow ball bouncing off any type of flat surface.

The results of this investigation support and extend those of *Air Pressure and the Coefficient of Restitution of a Ball,* by Osman and Kim in 2009[3]. Apart from the minimal difference regarding the limit approached by the coefficient of restitution with the increase in pressure, this investigation strongly supports the model proposed by Osman and Kim, even more so with a more precise investigation of a lower range of ball pressure.

References

[1] "Ideal Gas Law." *Wikipedia*. Wikimedia Foundation, 04 Aug. 2013. Web. 08 Aug. 2013.

[2] Osman, Kareem, and Boree Sophie Kim. "Air Pressure and the Coefficient of Restitution of a Ball." *ISB Journal of Physics*. International School Bangkok, June 2009. Web. 08 Aug. 2013. <www.isb.ac.th/HS/JoS>

[3] "Coefficient of Restitution." *Wikipedia*. Wikimedia Foundation, 04 Aug. 2013. Web. 08 Aug. 2013.