Resonance in Bottles with Different Shapes

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Abstract

The Helmholtz Resonance equation derived in the 1800's describes the nature of resonance in narrow-necked vessels, known as Helmholtz Resonators. It is commonly accepted that when air is blown across the opening of a bottle, the resonance can be modeled by the Helmholtz equation. Resonance was studied in two differently shaped bottles as the volume of the air cavity was varied. It was found that resonance in one of the bottles was accurately modeled by the Helmholtz equation but not in the other.

Introduction

When air is blown across the opening of an object with a spherical cavity and a sloping neck, known as a Helmholtz Resonator, the resonance is known as Helmholtz Resonance. Helmholtz Resonance is described by the equation

$$T = \frac{2\pi}{v}\sqrt{\frac{L}{A}} \bullet \sqrt{V} \qquad \text{Equation 1}$$

where T is the period, v is the velocity of the sound wave in the medium, L is the length of the neck, A is the cross-sectional area of the neck, and V is the volume of the air cavity.

Helmholtz resonance is an oscillating wave system. Essentially, when air is blown across the opening, the air in the neck flows inward and exerts a compression force inside the bottle. Once compressed, the air in the cavity then rebounds and flows out of the bottle, creating an environment inside the bottle with a pressure that is lower than that of the surroundings. Overcompensation causes air to then flow back into the bottle creating an oscillating system.

Helmholtz Resonance is commonly accepted as an accurate representation of all resonances in a cavity. This is claimed on both Wikipedia[1] and Vibrationdata[2]. Blowing across any bottle, including a water bottle or beer bottle, is said to be modeled by the Helmholtz equation. The validity of this claim is tested to determine if Helmholtz Resonance accurately models the resonance in two bottles of different cavity and neck shape.

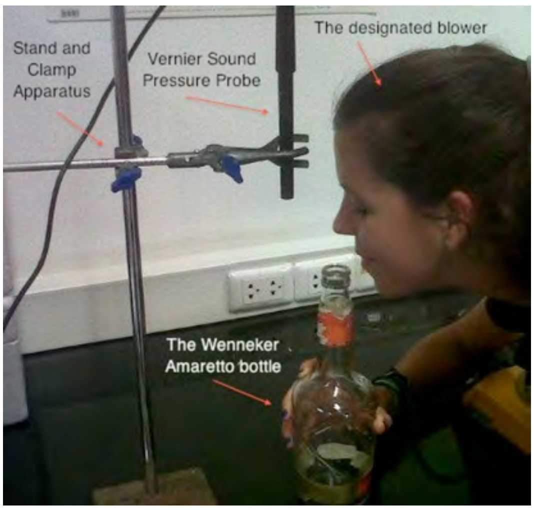

Figure 1 The experimental apparatus.

The openings of two differently shaped glass bottles were blown across to produce resonance, as shown in figure 1. The nature of this resonance was analyzed to determine whether it can be modeled by Helmholtz Resonance. If the equation does model the resonance in the glass bottles, a linear correlation between the period and the square root of the air cavity volume is expected (equation 1).

Methods

Air was blown across the opening of two differently shaped bottles (an Amaretto bottle and a Chang bottle), The ambient temperature was 26ºC, rising to 27ºC in the bottles. The Amaretto bottle was a 700ml Wenneker Amaretto Liqueur bottle, with a neck length of 11.7 cm, cross-sectional area of the opening of 3.1 cm^2, and body cavity volume of 616 ml, while the Chang bottle was a 660 ml Chang Beer bottle with a neck length of 6.5cm, cross-sectional area of the opening of 4.7 cm^2, and body cavity volume of 630 ml, as shown in figures 2 and 3. It is important to note that the definitions of the neck lengths are based on visual approximations from the opening to the shoulders of both the Amaretto and Chang bottles.

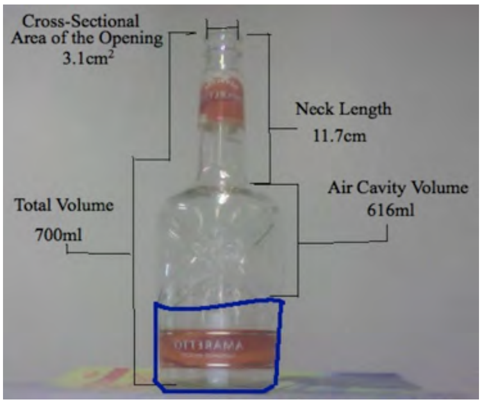

Figure 2 Amaretto bottle with labeled sections.

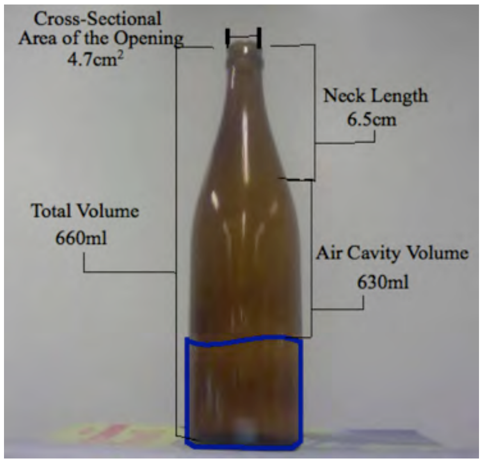

Figure 3 Chang bottle with labeled sections.

The resonance was recorded at 100,000 samples per second using a Vernier microphone and the frequency of the resonance was analyzed using a Fast Fourier Transform (FFT graph). Different amounts of water were added to the bottle to change the volume of the air cavity. The data was then analyzed to derive the relationship between the volume of the air cavity and the period of the resonance.

Results and Discussion

There is a linear relation between the period of the oscillation of the resonance and the square root of the volume of the air cavity when the Amaretto bottle is blown across. The equation representing this graph is

$$T = 0.33 \frac{ms}{\sqrt{ml}} \bullet \sqrt{V} - 0.42ms$$

Equation 2

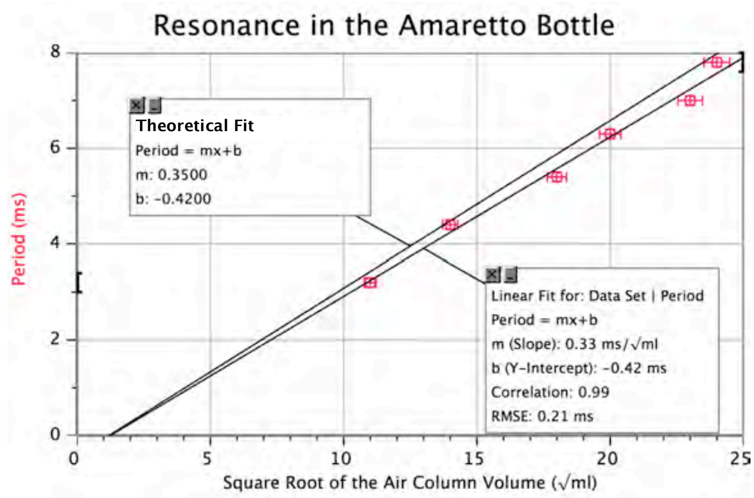

Figure 4 Period vs. √(Volume of air cavity) for the Amaretto Bottle. The slope of the line clearly demonstrates Helmholtz Resonance.

If this phenomenon is accurately modeled by the Helmholtz equation, then according to equation 1 the slope of the graph is

$$\frac{2\pi}{v}\sqrt{\frac{L}{A}}$$

Equation 3

By substituting the variables for the measurements of the bottle, the slope that is predicted for Helmholtz Resonance is calculated to be

$$T = 0.35\frac{ms}{\sqrt{ml}} \qquad \text{Equation 4}$$

In equation 1, the graph is expected to be proportional, though as can be seen in figure 4, the graph is linear, with the y-intercept at -0.42ms. This non-zero y-intercept indicates that the definition of the neck length is inaccurate, which is likely due to the fact that the lengths are based on arbitrary visual judgment. If the neck length were defined as slightly longer, all the air cavity volumes would decrease slightly, resulting in the fit going through the origin. The fact that the y-intercept is negative suggests that the effective air cavity volume is slightly smaller than the derived volumes used. The 6% difference between the theory and the measured values shows that the Amaretto bottle's resonance is accurately modeled by the Helmholtz equation.

When the Chang bottle is blown across, there is a linear correlation between the square root of the air cavity volume and the period of the wave produced. The equation representing this relationship is

$$T = 0.29\frac{ms}{\sqrt{ml}} \bullet \sqrt{V} + 0.49ms \qquad \text{Equation 5}$$

If this phenomenon is ideal Helmholtz Resonance, then the slope of the graph is, again, modeled by equation 4. By substituting the variables for the measurements of the bottle, the slope that would be expected for Helmholtz Resonance is calculated to be

$$T = 0.21\frac{ms}{\sqrt{ml}} \qquad \text{Equation 6}$$

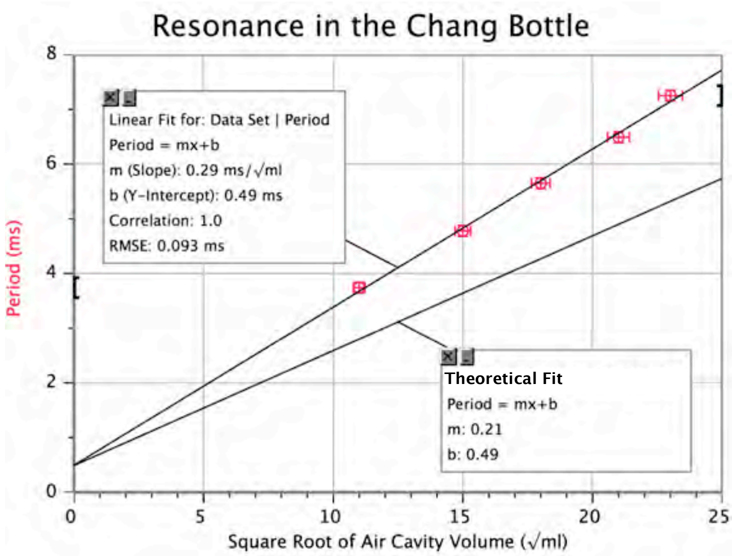

Figure 5 Period vs. √(Volume of air cavity) for the Chang Bottle. The slope of the line clearly does not demonstrate Helmholtz Resonance, whose relationship is shown in the theoretical fit line.

Like figure 4, figure 5 is linear, not the predicted proportional graph. The positive y-intercept suggests that the measured neck length is slightly greater than the effective neck length of the resonance. If the length were measured differently, then the fit would pass through the origin, but the slope would remain the same. The 26% difference between the theoretical fit and the experimental value of the slope shows that the Chang bottle resonance is not accurately modeled by the Helmholtz equation.

The reason the Amaretto bottle follows Helmholtz Resonance and the Chang bottle does not is unclear. There are differences in all the dimensions of the bottles, but it is suggested that the main cause is the difference in the shape of the slope of the shoulders of the two bottles.

It is suggested that the relationship between the slope of the shoulders and the period of the resonance be further researched to determine whether it is this factor that determines whether a bottle's resonance will be accurately modeled by the Helmholtz equation.

Conclusion

It has been shown that, contrary to what is commonly believed, not all bottles resonate according to the Helmholtz equation. The resonance in the Amaretto bottle is accurately modeled by the Helmholtz equation, while the Chang bottle resonance is not. The Amaretto bottle has a distinct body and neck, since the shoulders are almost horizontal. The division between the neck and the body of the Chang bottle is not clear, since the shoulders of the bottle are very sloped. The fact that the Chang bottle produced a resonance that could not be modeled by Helmholtz suggests that the shape of the shoulders of a bottle determines whether or not the resonance can be modeled by the Helmholtz equation.

References

[1] *Helmholtz resonance.* (n.d.). Wikipedia. Retrieved March 25, 2012, from http://en.wikipedia.org/wiki/Helmholtz_resonance
[2] Irvine, T. January 2004 Newsletter. VibrationData. Retrieved March 25, 2012, from http://www.vibrationdata.com/Newsletters/January2004_NL.pdf

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