Tube Length and Water Flow

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Abstract

In this study water flowed through a straight horizontal plastic tube placed at the bottom of a large tank of water. The effect of changing the length of tubing on the velocity of flow was investigated. It was found that the Hagen-Poiseuille Equation is valid when the effect of water entering the tube is accounted for.

Introduction

When water flows through a pipe, friction along the walls limits the velocity. For a constant head of water in the tank, the average velocity decreases with increasing tube length. For laminar flow of liquids through long cylindrical pipes, the Hagen-Poiseuille Equation[1] gives the pressure drop as the liquid flows down the pipe as

$$\Delta P = \frac{8\mu}{Q\pi r^4} \times L \qquad\qquad \text{(Equation 1)}$$

where ΔP is the change in pressure, μ is the dynamic viscosity of the liquid, r is the radius of the tube, L is the length of the tube, and Q is the volumetric flow. Since the tube is cylindrical, the volumetric flow rate in terms of velocity v is

$$Q = \pi r^2 v \,. \qquad\qquad \text{(Equation 2)}$$

Combining equations 1 and 2, the relationship between velocity and length of tubing is

$$v^{-1} = \frac{8\mu}{\Delta P r^2} \times L \,. \qquad\qquad \text{(Equation 3)}$$

Equation 3 is applicable to fluid flowing continuously in a tube. Since the still water in the tank must first enter the end of the tube, it is predicted that a constant must be included in the equation, as shown,

$$v^{-1} = \frac{8\mu}{\Delta P r^2} \times L + C \,. \qquad\qquad \text{(Equation 4)}$$

Since this equation applies to non-turbulent flow, it may not perfectly model the situation studied. For the head of water used in this investigation, the velocity of the water flowing

through the tube is relatively high, therefore some turbulence may be expected. Reynolds number[2] can be used to distinguish the type of flow

$$Re = \frac{QD_h}{vA}.$$
(Equation 5)

where D_h is the hydraulic diameter, v is the kinematic viscosity, and A is the pipe cross sectional area. For water flowing in a pipe, Reynolds numbers of less than 2300[3] are considered to be the laminar type, while turbulent flow begins at values of 4000 and above.

Method

Plastic tubing with a radius of 2.00 ± 0.05 mm was attached to a hole at the bottom of a water tank as shown in figure 1. The water tank was filled with water at $30\pm1°C$ to a height of 0.43 ± 0.01 m above the center of the tube, and positioned such that the tubing lay flat on the ground. Water was allowed to flow into the beaker for 30 ± 0.2 s and the volume of water collected measured. The pressure difference across the tube was kept constant by maintaining the water level at 0.43 ± 0.01 m. Three trials were performed for each of eight different lengths of tubing.
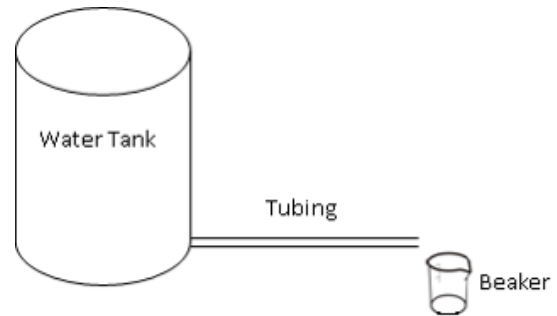
**Figure 1** The experimental apparatus.

Results and Discussion

It can be seen from figure 2 that the relationship between water flow rate and length of tubing is

$$v^{-1} = 0.37L \; \frac{s}{m} + 0.8 \; s$$
(Equation 6)

The velocity of the water flowing through the tube varies inversely with the length of tubing with a proportionality constant of 0.37 s/m. The theoretical value of the proportionality constant from equation four is 0.38 s/m, showing that the modified Hagen-Poiseuille Equation models this situation within uncertainties.

The flow rates used have Reynolds number ranging from 1,000 to 4,000. This suggests that the Hagen-Poiseuille Equation is reliable for predicting velocity values up to the limit of turbulent flow.

It should be noted that equation 5 is not applicable as the length of the tubing approaches zero. According to figure 2, the predicted velocity of water flowing through a hole in the side of the tank is the inverse of 0.8 s/m, or 1.25 m/s. This is inconsistent with the known value, just below 2.9 m/s. As the tube length approaches zero, the flow becomes turbulent, as the Reynolds number approaches 14,000, and equation 5 is no longer a valid model of the situation.
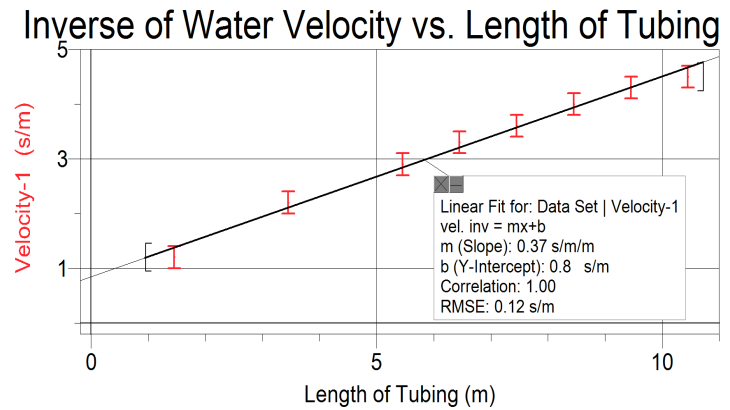
Figure 2 The inverse of the water velocity vs length of tubing.

It would be valuable to investigate situations involving greater pressure differential or shorter tubes to determine at what speeds the Hagen-Poiseuille Equation becomes invalid. Studying flow through tubes of different diameter would also be useful, in order to determine the relationship between tube diameter and the constant, C, from equation 5.

Conclusion

The relationship between the inverse of the velocity of water flowing in a tube and the length of the tubing is linear as described by equation 5. The results demonstrate the applicability of a modified Hagen-Poiseuille Equation to the situation of water flowing out of a tank through a long tube.

References

[1] *Hagen-Poiseuille Equation*. Wikipedia. May 24, 2011. Retrieved from
http://en.wikipedia.org/wiki/Hagen%E2%80%93Poiseuille_equation

[2] *Reynolds Number*. Wikipedia. July 22, 2011. Retrieved from
http://en.wikipedia.org/wiki/Reynolds_number

[3] *Reynolds Number*. The Engineering Toolbox. Retrieved from
http://www.engineeringtoolbox.com/reynolds-number-d_237.html