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Resonance and Neck Length for a Spherical Resonator

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Abstract

The relationship between the neck length of a spherical resonator and its period of fundamental resonance was investigated. This was done by measuring the frequency of fundamental resonance of the resonator at 6 different neck lengths. It was found that its resonance resembled Helmholtz resonance but was not that of ideal Helmholtz resonance.

Introduction

A classic Helmholtz resonator is a hollow, non-spherical cavity with a narrow opening through a tapered neck, as shown in Figure 1. When a stream of air is directed across the opening, an oscillation is set up in the cavity. Helmholtz resonance can be defined as the phenomenon of a vibrating plug of air in the neck of the resonator. The frequency of an ideal Helmholtz resonator is given by

Figure 1 A classic Helmholtz resonator.[1]

$$f_H = \frac{v}{2\pi}\sqrt{\frac{A}{V_o L}} \qquad \text{(Equation 1)}$$

where f_H is the Helmholtz resonant frequency, v is the speed of sound in the gas, A is the cross-sectional area of the neck, V_0 is the static volume of the cavity and L is the neck length.[2] Thus, Equation 1 can be re-written as

$$T_H^2 = \frac{4\pi V_o}{v^2 r^2} \times L \qquad \text{(Equation 2)}$$

where T_H is the period of Helmholtz resonance. It is known that the mass of the vibrating plug of air is greater than the mass of air contained within the neck. It is thus necessary to define an effective neck length, which, for Helmholtz resonators with round necks, has been shown to be

$$L_{eff} = L + 1.5r \qquad \text{(Equation 3)}$$

where L_{eff} is the effective neck length and r is the radius of the neck.[3] Given the area of a circle, the period of a Helmholtz resonator becomes

$$T_H^2 = \frac{4\pi V_o}{v^2 r^2} \times L + \frac{4\pi V_o}{v^2 r^2} \times 1.5r \qquad \text{(Equation 4)}$$

When the neck length of an ideal Helmholtz resonator is plotted against its period of fundamental resonance squared, it is expected that the relationship formed will be linear with a positive slope equaling $((4\pi V_0)/(v^2 r^2))$ and a y-intercept equaling $((4\pi V_0)/(v^2 r^2)) \times 1.5r$.

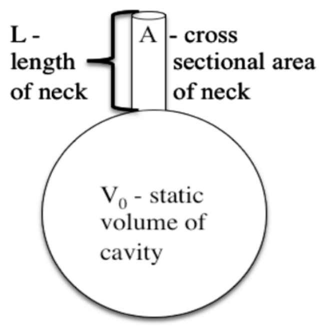

Figure 2 Spherical resonator with cylindrical neck.

In this investigation, the relationship between the neck length of a spherical, Helmholtz resonator with a cylindrical neck and its period of fundamental resonance is studied to determine if this behaves like the classic Helmholtz resonator with the tapered neck.

Methods

A spherical, plastic ball with a volume of 180 ± 3 ml, an inner diameter of 0.0050±0.0005 m and a skin thickness of 0.25±0.01 mm was used in this investigation. A circular hole was made in the ball through which a straw (the neck) was inserted.

Air was softly blown over the straw to create the fundamental resonant frequency. The sound was recorded by a microphone which was connected to LoggerPro and set to collect 100,000 samples per second for 0.5 seconds. An FFT was performed to determine the fundamental frequency of the sound. Six neck lengths were used, ranging from 0.87 mm to 4.97 mm. Straw lengths were measured using a Vernier caliper. Three trials were conducted at each neck length.

The external temperature was 26±2°C for the duration of the investigation while the internal temperature of the plastic ball while blowing was measured to be 31±2°C. The uncertainties for the calculated values were found using the original instrumental uncertainty as well as the range of the data collected.

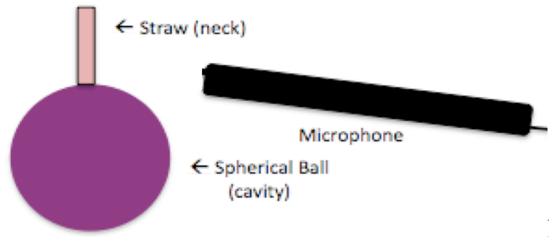

Figure 3 Diagram of experimental apparatus.

Results and Discussion

As shown by Figure 4, the relationship between the neck length of the resonator and its period of fundamental resonance can be expressed ±as

$$T_H^2 = \left(290 \pm 40 \times 10^{-5} \frac{s^2}{m}\right) L + (2 \pm 1 \times 10^{-5}\ s^2) \qquad \text{(Equation 5)}$$

indicating that the relationship is positively linear as expected. It is important to compare the values proposed by the experimentally derived relationship to those proposed by theory. The theoretical slope value, as calculated using Equation 4, is 290 ±60 x$10^{-5}$$s^2$/m and the theoretical calculated y-intercept value is (1.1±0.4)x10^{-5} s^2. While the experimental and theoretical relationships are consistent within uncertainties, the validity of Equation 5 as a model for the behavior of the resonator is questioned due to the large uncertainties and poor quality of fit of the experimentally derived relationship.

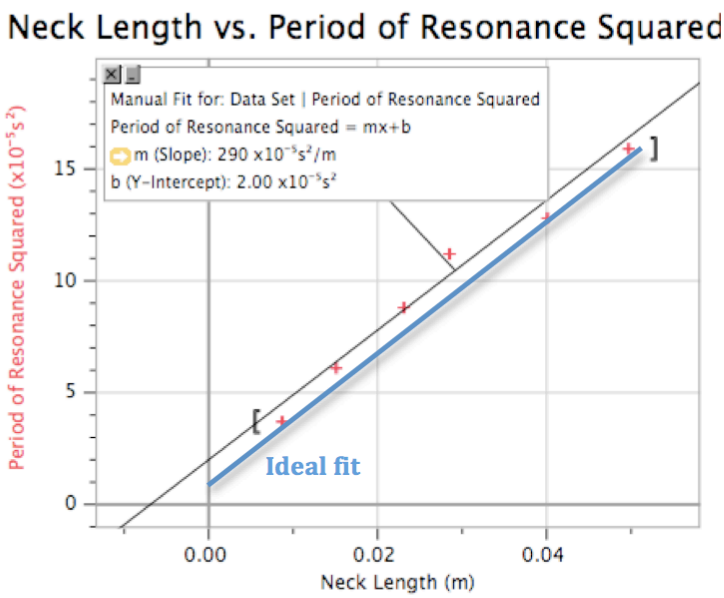

Figure 4 The neck length of the resonator plotted against the period squared along with the line of best fit, and the expected line calculated using the Helmholtz relationship (Equation 5).

The difference in the y-intercepts of the two equations suggests that the $1.5r$ adjustment to the effective neck length proposed by Equation 3 might not be valid for this situation. An adjustment of $2.7r$ to the effective neck length is predicted by the data.

It is important to note that the three trials of resonant frequencies recorded for each neck length were the same, within instrumental uncertainty, suggesting that each data point is very reliable. This limits the confidence with which the experimentally derived ideal Helmholtz relationship (Equation 5) can be proposed since the experimentally derived fit misses many of the data points by a considerable margin. An alternate fit proposed in Figure 5, which only takes into account the first four data points, can be expressed as

$$T_H^2 = \left(370 \pm 10 \times 10^{-5} \frac{s^2}{m}\right) L + (0.4 \pm 0.2 \times 10^{-5}\ s^2) \qquad \text{(Equation 6)}$$

The quality of fit of Equation 6 within the domain of the first four data points is strong. This suggests the possibility that the phenomenon of resonance within a spherical Helmholtz resonator with short neck lengths does not follow ideal Helmholtz resonance. It also suggests that the model changes with changing neck length. Further investigation, however, is needed to come to a more reliable conclusion.

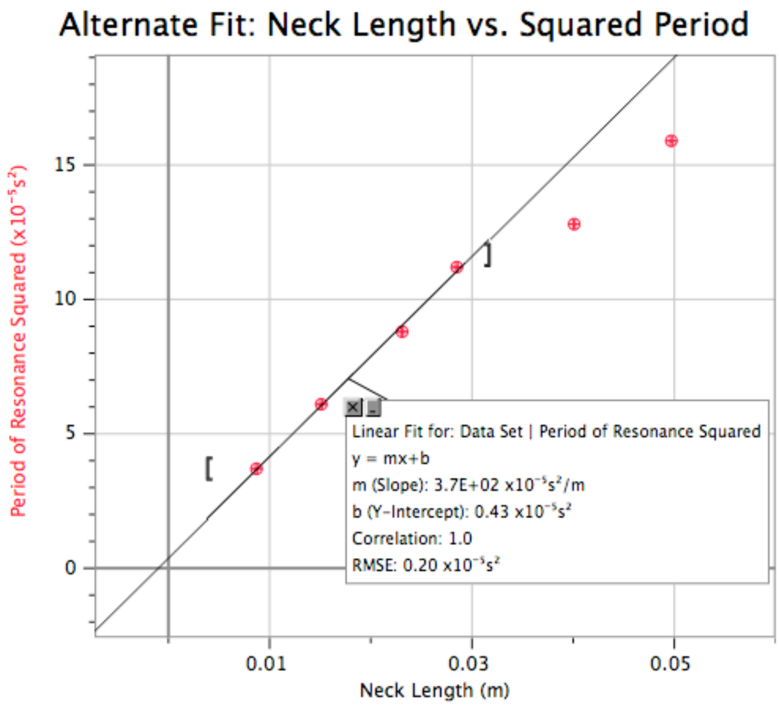

Figure 5 Shows the neck length of the resonator plotted against its period of fundamental resonance squared with an alternate fit which includes only the first four data points (Equation 6).

A weakness which limits the overall confidence in the results of the investigation is the variation in the internal temperature of the resonator throughout the investigation (from 29 °C to 33 °C). A possible improvement would be to use a pump to produce the air stream as the temperature of the pumped air would be at room temperature.

Further research could include investigating the relationship between the cross-sectional area of the neck of a spherical Helmholtz resonator and its period of resonance, further investigating the phenomenon of effective neck length in order to better define it mathematically, and further investigation the possible difference between the behavior of resonators with short versus long neck lengths.

Conclusion

It has been shown that the resonator studied in this investigation behaves within the confines of ideal Helmholtz resonance as proposed by theory. It is suggested that the definition of the effective neck length for a Helmholtz resonator with a round neck as L + 1.5r does not apply to the resonator being studied. It is also suggested that the model for resonance within a spherical resonator with short neck lengths might differ from that with long neck lengths.

References

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