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Helmholtz Resonance in a Water Bottle

A. Balachandran, F. Watanabe, and J. Lim

Abstract

The resonance that occurs when blowing across the top of a water bottle filled with different volumes of water was studied. It was shown that, contrary to popular belief, a water bottle is not an ideal Helmholtz resonator. Resonance in a water bottle with an extendable neck was then studied to determine how the length of the neck affects the resonance. The results showed that ideal Helmholtz resonance occurs when the neck length was in a middle range, while for no neck a standing wave resonance occurs. For a very long neck the results were inconclusive.

Introduction

When air is blown across a water bottle, the turbulent air flow across the open bottle interacts with the confined air and sets up an oscillation in the neck at a resonant frequency. Known as Helmholtz resonance, this phenomenon occurs when air is blown across containers with a wide range of cavity and neck shapes. For a round bottomed flask and assuming ideal conditions, it has been shown that resonance occurs at a frequency given by[1]

$$f = \frac{v}{2\pi}\sqrt{\frac{A}{VL}} \qquad \text{(Equation 1)}$$

where v is the speed of sound in air, A is the area of the neck, V is the volume of the cavity, and L is the length of the neck.

Equation 1 can be rearranged to show that for ideal Helmholtz resonance the period of resonance (T), is proportional to the square root of the volume of the cavity,

$$T = \frac{2\pi}{v}\sqrt{\frac{VL}{A}}. \qquad \text{(Equation 2)}$$

The first part of this investigation will determine whether the common assertion that blowing across a water bottle produces ideal Helmholtz resonance is valid[3,4].

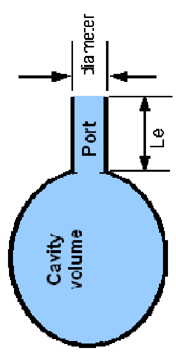

Figure 1 An ideal Helmholtz resonator.[2]

The effect of extending the neck of the bottle and how the length of the neck affects the extent to which the Helmholtz resonance model is applicable will also be investigated. For resonance at a given frequency, the relationship between the height of the air column in the water bottle (h) and the length of the neck (L) of an ideal Helmholtz resonator can be shown to be

$$\frac{1}{h} = \frac{4\pi^3 r^2 f^2}{Av^2} L \qquad \text{(Equation 3)}$$

where r is the radius of the bottle. Thus, if the water bottle is an ideal Helmholtz resonator, it is expected that the neck length will be inversely proportional to the height of the cavity for a given resonant frequency.

Methods

Air was blown across the neck of a standard 600 ml plastic drinking water bottle and the sound produced was recorded at 100,000 samples per second using a Vernier microphone. The frequency of the note was calculated using a Fast Fourier Transform. This process was repeated for several different levels of water in the bottle. The data was then analyzed to determine the relationship between air column height and frequency.

In the second part of the investigation, a tank was filled with water, and the bottom of a water bottle was cut off. A neck extension was made from plastic sheet and fitted into the neck of the bottle, as shown in figure 2. A 261.0 Hz tuning fork was struck and held above the neck of the water bottle, and the bottle was raised or lowered in the tank to find the position at which maximum resonance occurred. The distance from the surface of the water to the base of the neck was then recorded. This was repeated for neck lengths ranging from 0.170 m to 0.000 m, when the neck of the bottle was cut off. Neck length was measured from the base of the neck of the bottle. The bottle was then cut part way down the curved part and then at the base of the curved part, leaving a cylindrical tube, and the resonant length recorded for each. The temperature during the experiment ranged from 26 to 28°C.

Figure 2 The water bottle with extended neck and bottom cut off was adjusted to find the position of maximum resonance.

Results & Discussion

As stated previously, blowing across a water bottle is often claimed to demonstrate Helmholtz resonance. However as shown in Figure 5, the data appears to follow a power relation with exponent 0.75, rather than the square root relationship predicted from Equation 1.

This clearly demonstrates that the resonance produced by blowing across a water bottle is not ideal Helmholtz Resonance. Rather, the relationship between period and air column height for a water bottle is shown to be

$$T = (0.014\ s/m)\ h^{0.75} \qquad \text{(Equation 4)}$$

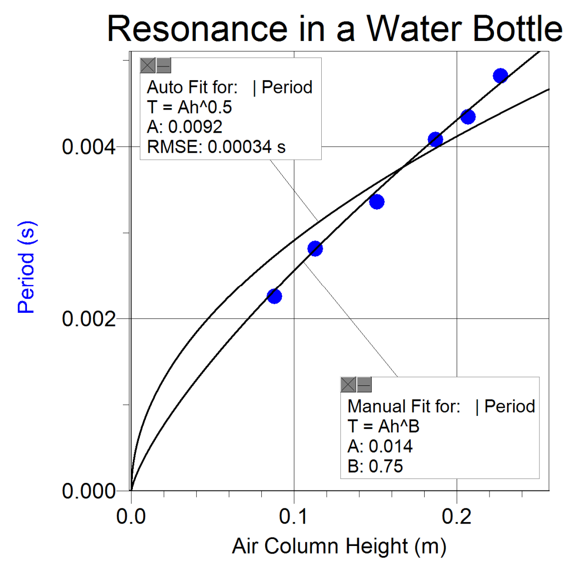

Figure 3 Period vs air column length for a normal water bottle. The square root relationship predicted by Helmholtz resonance does not fit the data.

Figure 4 shows the results for the second part of the investigation, in which the relationship between neck length and air cavity height at which maximum resonance occurred was determined for a given resonant frequency. The results suggest that for neck lengths ranging from 0.025 m and 0.170 m, the length of the neck is inversely proportional to the height of the air cavity at resonance. For this range, the relationship between neck length and cavity height can be modeled by the equation

$$1/h = (121 m^{-2})\ L + 3m^{-1} \qquad \text{(Equation 5)}$$

According to equation 2, the slope of figure 4 is expected to equal $\frac{4\pi^3 r^2 f^2}{Av^2}$ or, using measured values, 125 m^{-2}. The difference of less than four percent supports the idea of ideal Helmholtz resonance occurring in the water bottles with neck lengths in this range.

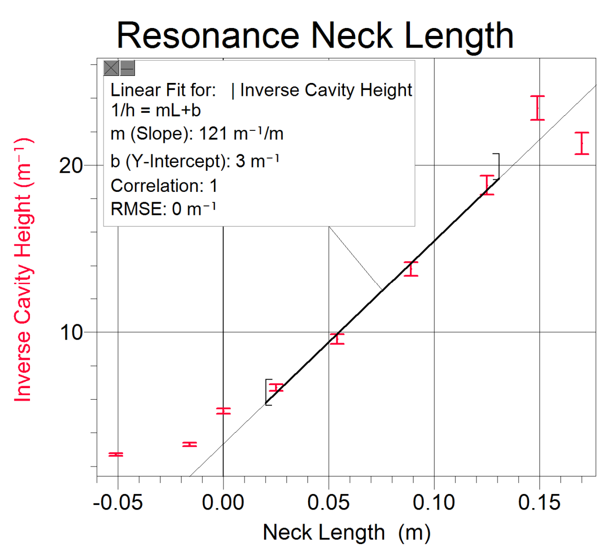

Figure 4 Neck length versus inverse cavity height at resonance. For neck lengths between 0.025 and 0.176 m the linear fit suggests Helmholtz resonance.

As the top of the bottle was cut off, shown in figure 4 as negative neck lengths, the cavity approached a cylindrical shape. Under these conditions, the air column no longer behaved as a Helmholtz resonator, but rather a standing wave was set up in the column. Knowing that a standing wave of one-quarter wavelength is formed in a cylindrical air column which is open at one end,[5] the predicted cavity length of maximum resonance was 0.331 m, compared to the measured value of 0.314 m when there was no curved portion left on the bottle. This suggests that a standing wave was set up in this case, while the bottle with the neck cut off and with the curved portion partly cut off involve a mode of resonance which is transitional from standing wave to Helmholtz resonance.

The reason for the longest two neck lengths not following the trend is unclear. The upper limit on neck length for Helmholtz resonance is not known, so it is possible that a non-ideal Helmholtz resonance was occurring at these extremely long neck lengths. However, the level of confidence in these two data points is quite low because was difficult to identify the point of resonance for these neck lengths, as the resonance amplitude was very low, so.

It is suggested that further research be conducted to determine what cavity shapes lead to ideal Helmholtz resonance. The maximum neck length at which Helmholtz resonance occurs for a spherical cavity Helmholtz resonator could also be investigated.

Conclusion

It has been shown that, contrary to what is commonly claimed, a 600 ml water bottle does not behave as an ideal Helmholtz resonator when it is blown across. However, for extended neck lengths there is a range for which the bottle behaves like an ideal Helmholtz resonator

References

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