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Water Oscillation in an Open Tube

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Abstract

When an open tube is placed in a tank of water, covered on top, raised, and then uncovered, the water inside the tube will oscillate. The characteristics of the oscillation of the water inside the tube were studied. It was shown that, for large oscillations, the top half-period was longer than the bottom half period due to the increased mass of the water column. For small oscillations, it approached simple harmonic motion, with the square of the period varying with mean length, as predicted by theory. An end correction was also shown to exist, due to the motion of the water outside the bottom of the tube during the oscillation. The end correction was shown to be independent of the mean length of the water column, as predicted.

Introduction

A simple harmonic motion is an oscillation for which the acceleration is proportional to the displacement. An example is the oscillation of mass on a spring that obeys Hooke's law. The period of a simple harmonic motion is independent of the amplitude. A simple harmonic oscillator is illustrated in figure 1. The period of oscillation and the amplitude remain constant throughout the oscillation.

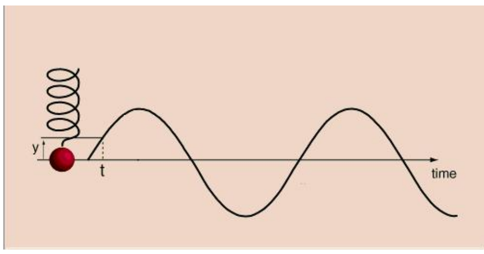

Fig 1: Simple Harmonic Motion

Damped simple harmonic motion occurs when a dissipative force opposes the motion, removing energy from the oscillation. Damping may be due to friction or air resistance. To a very good approximation, the period of damped simple harmonic motion remains the same as the amplitude gradually decreases. The motion of a damped simple harmonic oscillator is illustrated in figure 2.

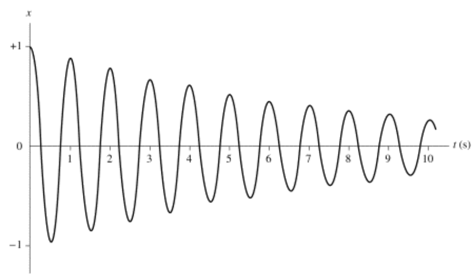

Fig 2: Damped Simple Harmonic Motion

The equation for the period of a mass oscillating on a spring for damped and undamped motion is the same. The formula for the period is …

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad \text{(Equation 1)}$$

… where m is the mass of the object oscillating and k is the spring constant (or force constant).

When a cylindrical tube, open on both ends, is placed into water, oscillations of the water column inside the tube can be induced by closing the top end of the tube, raising the tube, then opening the end, as shown in figure 5. It is expected that the motion of the oscillating water in the tube will be simple harmonic motion. The period is expected to depend on the mass of the water and the acceleration due to gravity as follows.

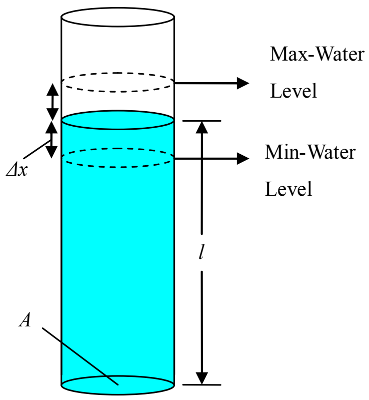

Fig 3: Water oscillation in a tube

The restoring force for this oscillation is the weight of the water above or below the equilibrium position. $F = -k(\Delta x)$ is the expression for the restoring force for any simple harmonic oscillation where a = acceleration of water, $-\Delta x$ = negative displacement of water from equilibrium level, m = mass of water, k = force constant. Since $F = ma$, $ma = -k(\Delta x)$, hence…

$$\frac{m}{k} = -\frac{\Delta x}{a}. \qquad \text{(Equation 2)}$$

From figure 3, the mass of the water in the tube m is, $m = lA\rho$, where A is the cross sectional area of the water column, ρ is the density of water and l is the length of water column. Likewise, $-\Delta x A\rho$ is the difference between the mass of water when it is at the equilibrium point of oscillation and the mass of water when it is at the minimum level (indicated by dotted lines). Hence, $F = ma = lA\rho a$ and $F = -\Delta x A\rho g$ which gives

$$la = -\Delta x g\,. \qquad \text{(Equation 3)}$$

If equations 2 and 3 are combined, $\frac{l}{g} = \frac{m}{k}$. When we substitute this to equation 1, the period of the water oscillating in a tube is ...

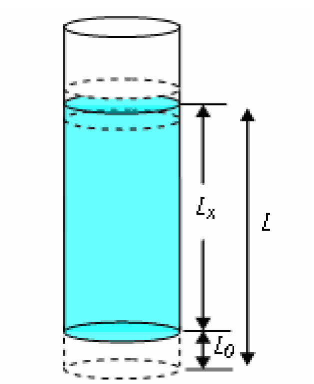

Fig 4: Water Tube and End Correction

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad \text{(Equation 4)}$$

… where l is the length of the water column and g is gravity.

From equation 4, one expects that for large oscillations the half-period for the top half and the bottom half of the oscillation will vary. This is because the displacement of the water level, hence the length of the water inside the tube will be significantly larger for the top half-period than for the bottom half-period. For small oscillations the length of the water column varies less; therefore it is expected that the water oscillation may be approximated by simple harmonic for small oscillations. Under the condition of a small oscillation, one could find the length of the water movement below the tube. Because the cylindrical tube is open ended, it is expected that the effective mass of oscillating water will be more than the actual mass enclosed in the tube because there will be an interaction between the oscillation and the water outside the open end. The predicted equation for the determination of the end correction of the water oscillation in a cylindrical tube can be derived as follows.

From the diagram, the actual length of water oscillating l is given by, $l = l_x + l_0$, where l_0 = length of end correction, l_x = depth of tube under water. If this is substituted to equation 4, and rearranged for l_0, l_0 is …

$$l_0 = g\left(\frac{T}{2\pi}\right)^2 - l_x . \qquad \text{(Equation 5)}$$

This equation implies that the end correction is independent of l_x (the initial depth of the tube into the water before oscillation). This is expected because the water under the tube is only affected by the movement at the end of the tube, not by the entire mass of the water in the tube.

To find the end correction of water oscillation in a cylindrical tube, it must be demonstrated that the motion of water approaches a damped simple harmonic motion for small oscillations. This can be shown by analyzing the half periods of an entire oscillation; from large oscillations to small oscillations (motion where the displacement is within 0.02m may be considered as small oscillation). When it is demonstrated that the motion is approximately damped simple harmonic for small oscillations, then one can proceed with calculating the end correction. Small oscillations can be observed for different equilibrium water column lengths (l_x) (figure 4). Then using equation 5, and the period and equilibrium depths (l_x) for each trial, the end correction can be calculated.

Methods

The procedures of this experiment involved two parts; Firstly measuring oscillations of varying amplitude to determine whether the oscillation of water in a tube is damped simple harmonic motion; Secondly measuring small oscillations with the tube at varying depths.

Half Periods and Simple Harmonic Motion

A water tank with height 0.815m was filled with water. A 50 mm diameter, transparent, cylindrical tube, open at both ends, was put into the water. A small Styrofoam ball was

Fig 5: A single frame showing the water in the tube and the tank

placed in the tube, to make it easier to track the oscillation on the video. The tube was held vertically so that approximately 1/4 of its total length was under the water. With one hand blocking the top end of the tube, the tube was pushed down in to the water. The hand was then released; the oscillation was recorded using a camera.

The procedure was repeated for oscillations in the opposite way; the tube was let to stand vertically at the lowest possible point in the water tank. With one hand blocking the top end, the tube was lifted up until 3/4 of its length was out of the water. The hand was then released and the motion of the water level was recorded on the camera (figure 6).

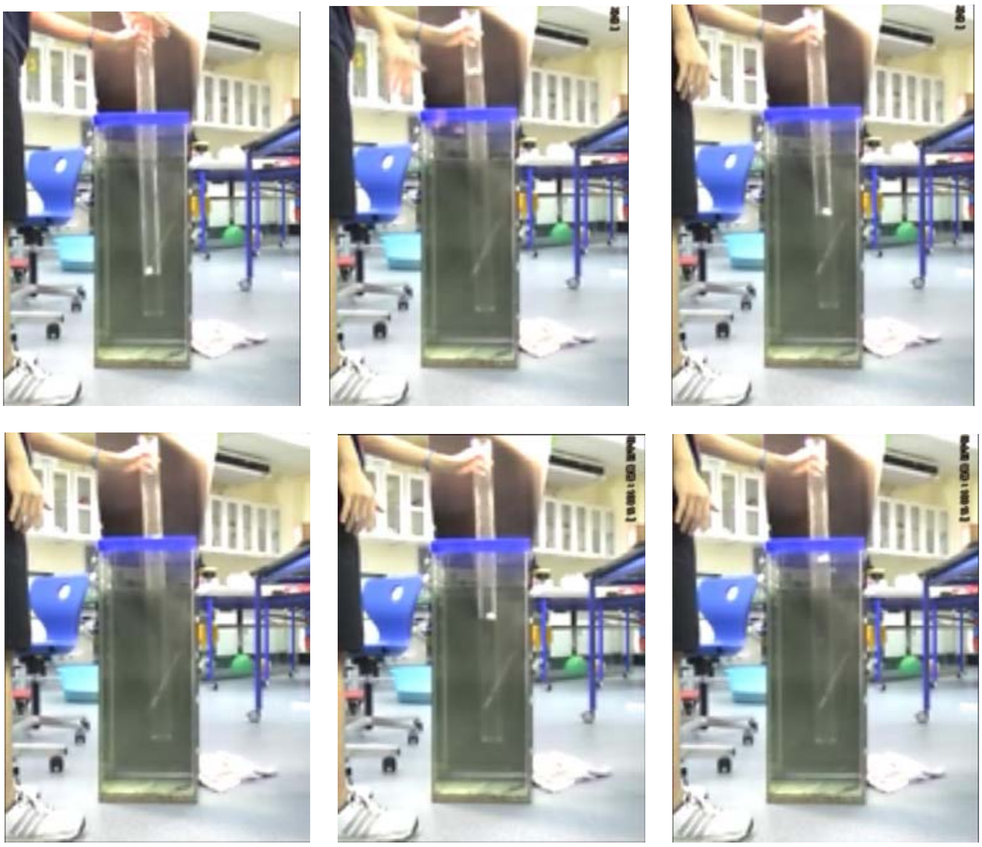

Fig 6: Screen Shots of Water Oscillation

End Correction

The previous procedure was repeated for small oscillations. A meter stick was attached to the side of the tube to indicate how deep the tube went into the water. To create small oscillations, the initial displacement (length to which the tube was pushed down from the top) was controlled to 0.02m. The oscillation recordings were started out with the tube at shallower depths, 0.045m, to deeper depths, 0.465m. The period of each motion was calculated by taking an average of time of 4-6 consecutive periods. The end corrections were found using the data of the periods and equation 5.

Results and Discussion

Half Periods and Simple Harmonic Motion

A line was fitted manually to indicate the equilibrium point of the oscillation. Appropriate linear fits were made along the graph to find the intersection points between the oscillation data and the manual fit. Using "interpolate" in logger pro, the value of the time at each intersection was found. The difference between the time value of each consecutive intersection was found and plotted on a graph as shown in figure 8.

Figure 8 shows that, for large oscillations, the half-periods are greater for the top half of the oscillation than for the bottom half, as predicted. As the oscillations become smaller, the difference in time for the top and bottom half periods is reduced, as expected. Finally, it can be seen that the time for one complete oscillation remains constant at 1.4 ± 0.1 s. The oscillation in the tube is simple harmonic motion to a good approximation. The quality of the data does not allow an analysis of whether the length of the half-periods as a function of amplitude and column length can be modeled on equation 4.

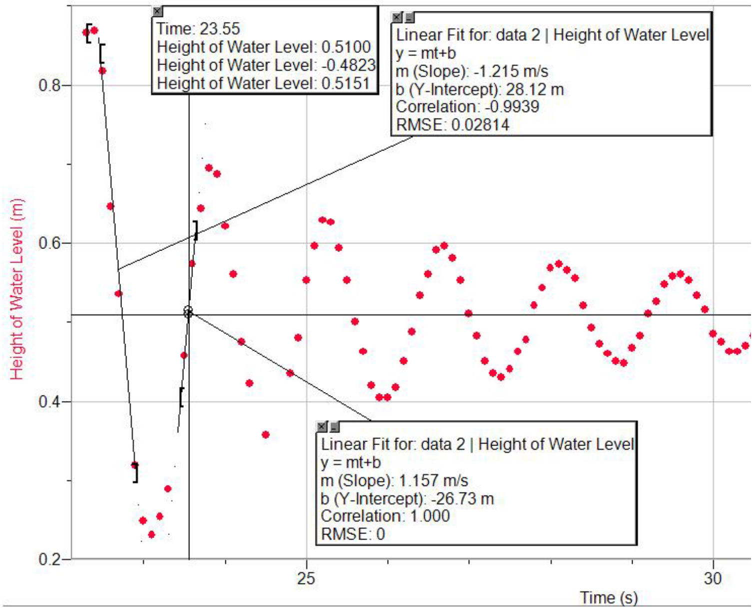

Fig 7: Oscillation of water in a Tube. The data was used to calculate half periods and whole periods as the oscillation was damped.

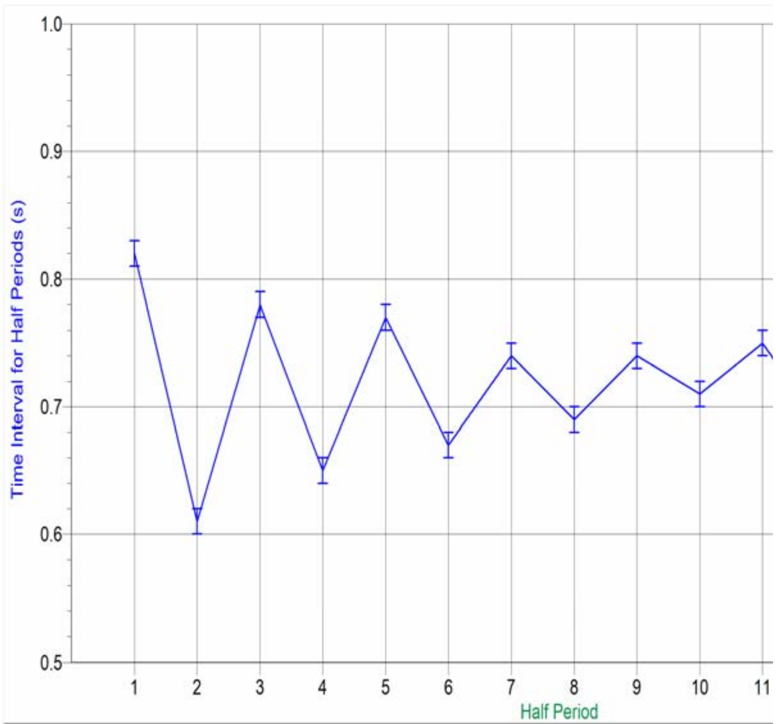

Fig 8: Time Interval between Half Periods. The difference in half periods reduces with amplitude, while the whole period remains constant.

End Correction

The period of the oscillation for small amplitude oscillation as a function of the median length of the water column in the tube was analyzed. The results are shown in figure 9.

The graph shows a linear relationship ...

$$y = mx + b$$

... where y = length of the water column in the tube (l_x in figure 4) , x = period squared, m = 0.248 m/s^2, and b = -0.017 m.

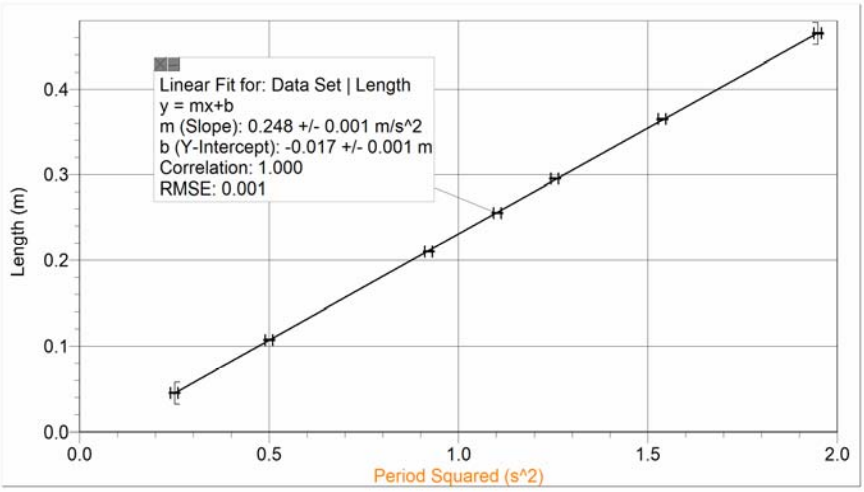

Fig 9: Length of mean water column versus Period Squared for small amplitude oscillations. The relationship is linear within uncertainty, showing that the oscillation is simple harmonic motion and the end correction is independent of column length, as predicted by equation 6.

If equation 5 is rearranged, it gives···

$$l_x = \left(\frac{g}{4\pi^2}\right)T^2 - l_0 . \qquad \text{(Equation 6)}$$

Thus, $\frac{g}{4\pi^2}$ is the slope, and $-l_0$ is the y-intercept b in figure 9.

If values of g = 9.8 and π = 3.14 are used, $\frac{g}{4\pi^2} = 0.248489 \approx 0.248$ m/s^2, showing that for small oscillations, equation 6 is a good approximation of the motion of a water column in a tube. The prediction of an end correction is supported. The end correction is shown to be 0.017 m for this tube, and is shown to be independent of the length of the water column.

Conclusion and Evaluation

The results showed that the water oscillation in a cylindrical tube is not a damped simple harmonic motion when the displacements are large. The half-period for the top half is significantly larger than for the bottom half due to the greater mass of the water column during the top half of the oscillation. Although this does not imply simple harmonic motion, it was shown that the whole period remains relatively constant for large amplitude oscillations. The results show that the oscillation can be approximated as damped simple harmonic motion for small oscillations, as described by equation 4. In figure 9, the square of the period is shown to vary proportionally with the length of the water column. It is also demonstrated that the end correction is independent of the mass of the water oscillating in the tube, and that end correction can be modeled by equation 6. For the specific tube used in this investigation, the end correction was determined to be 0.017 m.

There were two major weaknesses in the procedure which reduce confidence in the conclusions. Firstly, when the tube was covered and raised out of the water, it was held by hand, rather than being clamped in place. Because the observation of the water motion went over a prolonged period, for example, 40 seconds, the position of the tube may have changed due to hand vibration or other body movement; the tube may have gone into the water deeper, or the tube may have been tilted to an angle. Secondly, the quality of the video made it difficult to accurately track the oscillation of the water. A high-speed video with higher resolution would allow more precision in determining the position of the water level over time as it oscillated. This would also make it possible to determine if the difference in half-periods for large amplitude oscillations could be modeled by the proposed theory.

Further research could be conducted into the affect of the diameter of the tube on the end correction. The affect of the density and viscosity of the liquid on the period and end correction could also be investigated.

References

Simple Harmonic Motion. R. Nave, HyperPhysics Project, Georgia State University Retrieved January 14, 2008, from HyperPhysics: http://hyperphysics.phy-astr.gsu.edu/hbase/shm.html

Simple Harmonic Motion. Department of Physics, University of Guelph. Retrieved January 14, 2008, from: http://www.physics.uoguelph.ca/tutorials/shm/Q.shm.html